Manifold Sampling for Composite Nonconvex Nonsmooth Optimization

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Problem setup

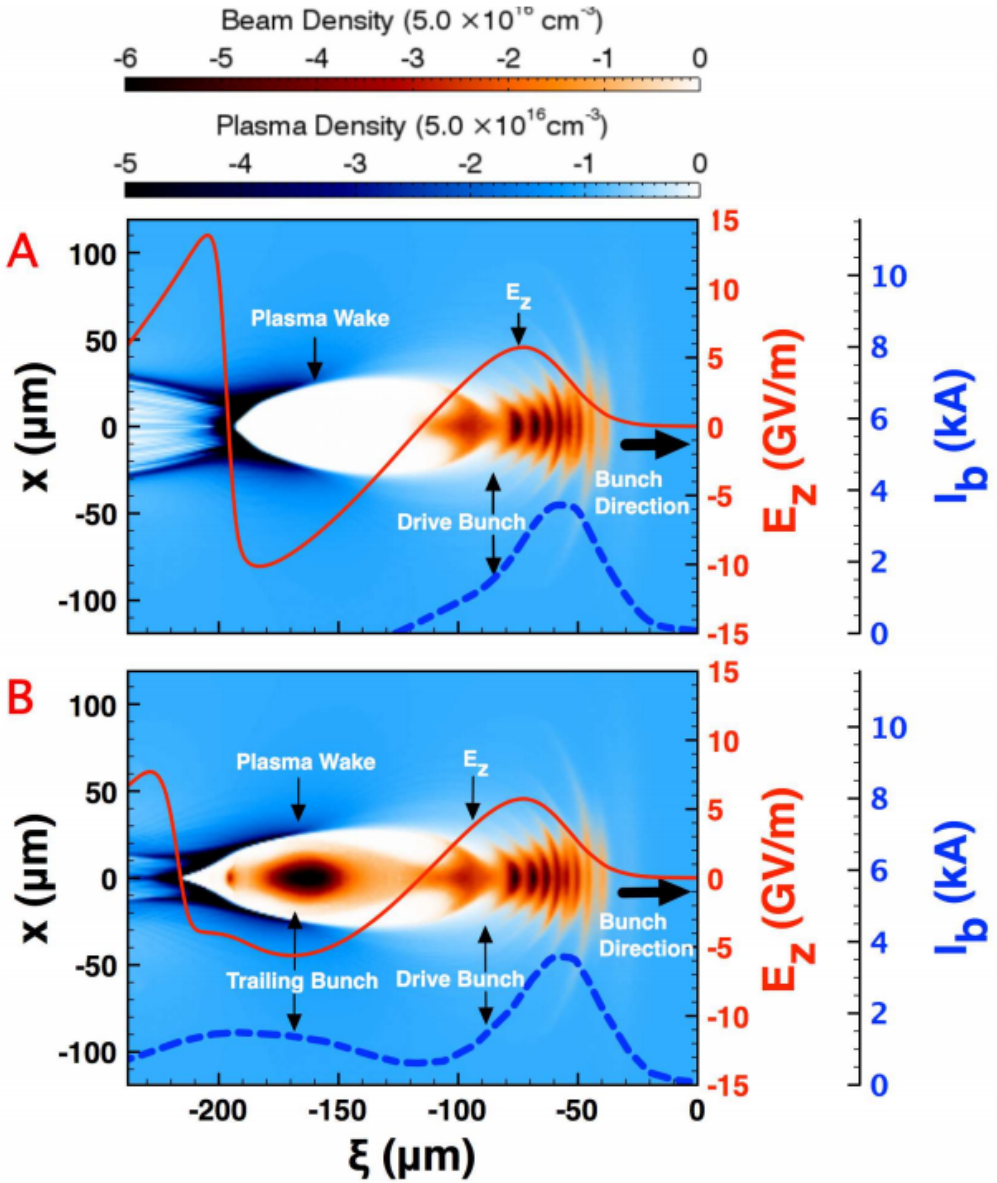
- Nonsmooth, composite optimization

$$\underset{x}{\text{minimize}}\, f(x) = h(F(x))$$

  nonsmooth $h\colon \mathbb{R}^p \to \mathbb{R}$ (with a known structure), smooth $F\colon \mathbb{R}^n \to \mathbb{R}^p$ (expensive to evaluate).
- Idea: Build p models, one for each component of F. Use model gradients in place of ∇F.
- Requires a *manifold representation* of h.
- Example: censored loss:

$$f(x) = \sum_{i=1}^{p} |d_i - \max\{c_i, F_i(x)\}|$$

Computers/Simulations!



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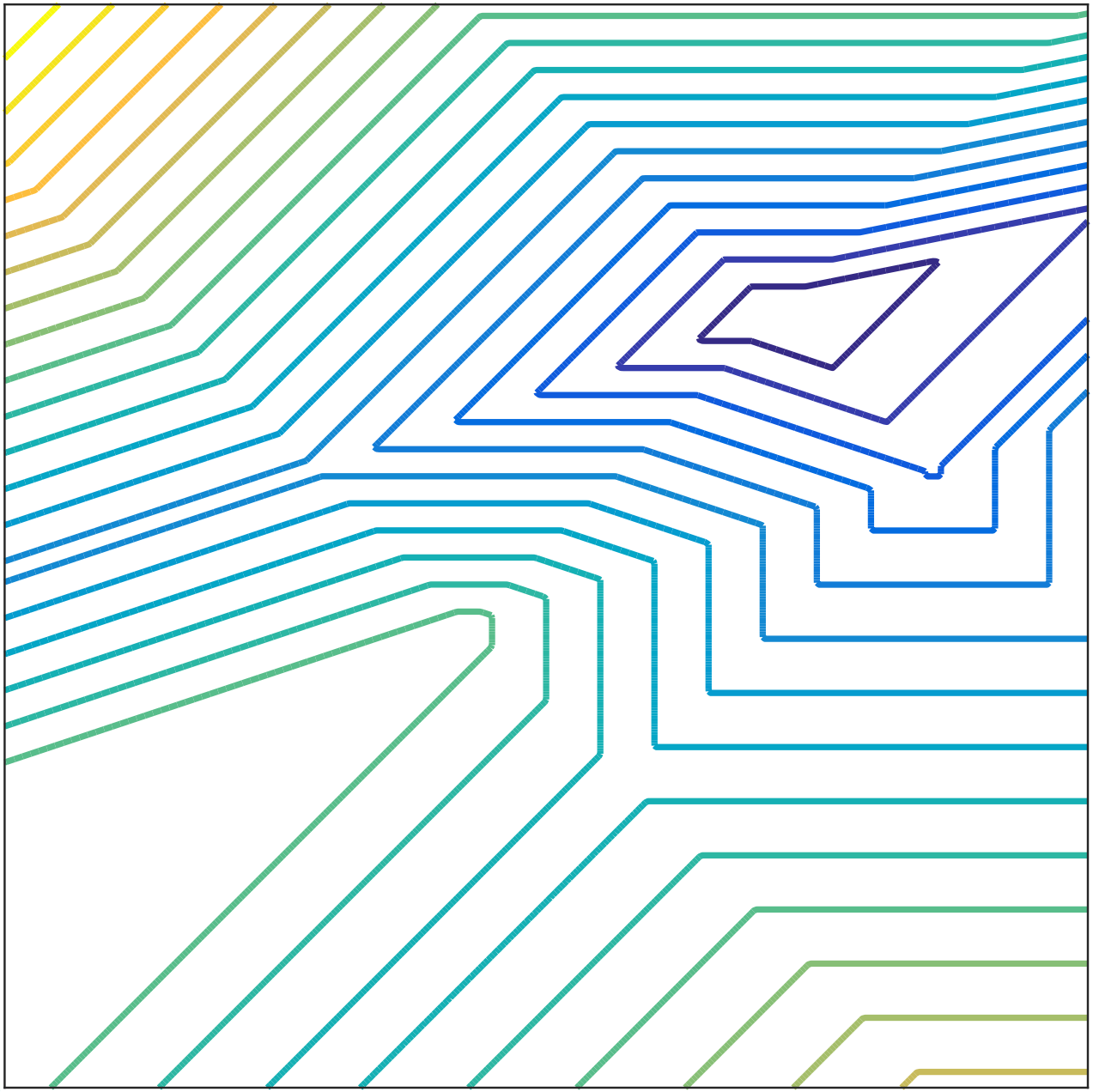

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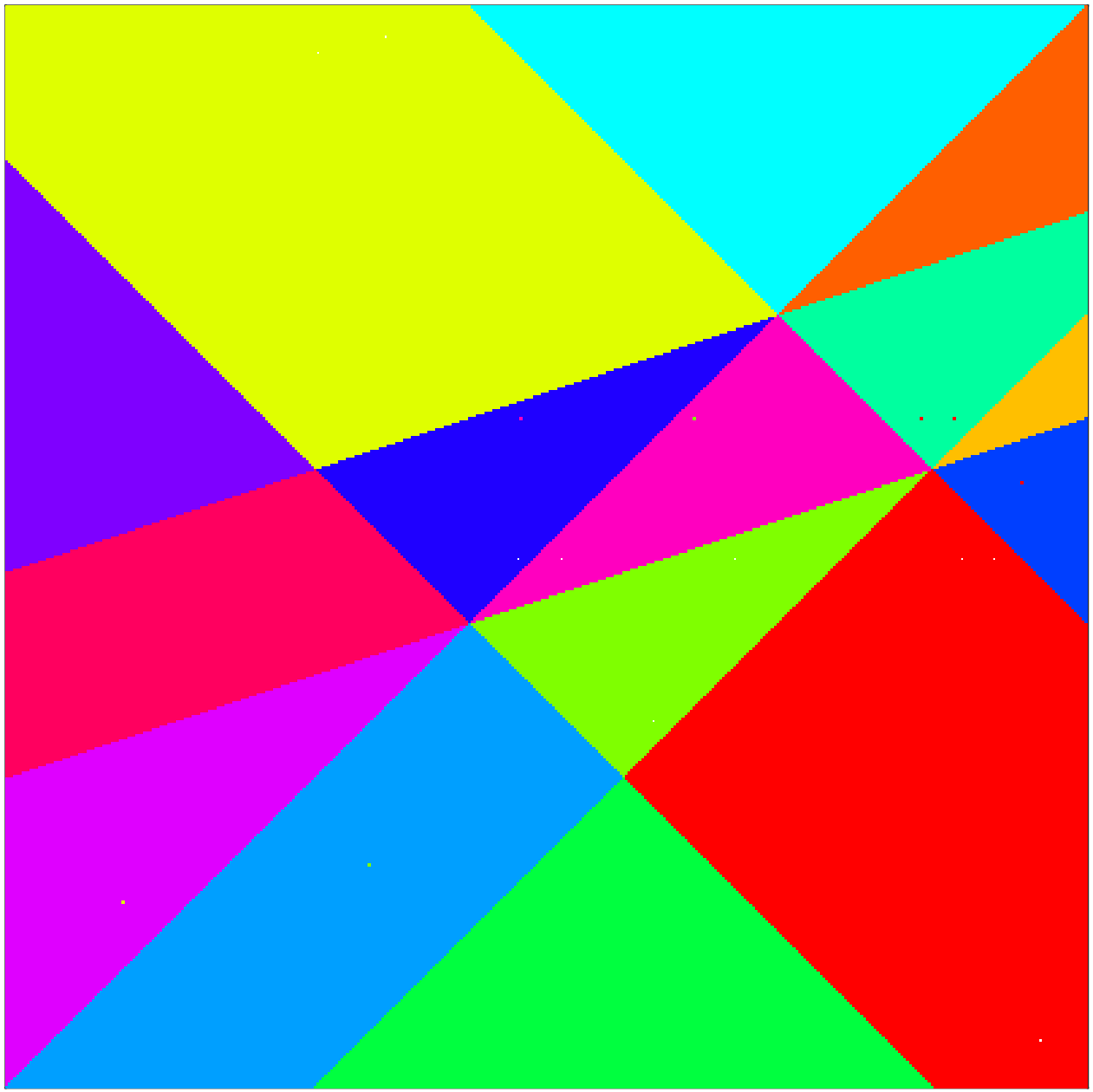

Manifold representation

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User scripts need to calculate:

$f(x)$, $F(x)$, $\mathbb{H}(F(x))$, $\{\nabla h_i(F(x)) : i \in \mathbb{H}(F(x))\}$, $\{h_i(F(x)) : i \in \mathbb{G}\}$,

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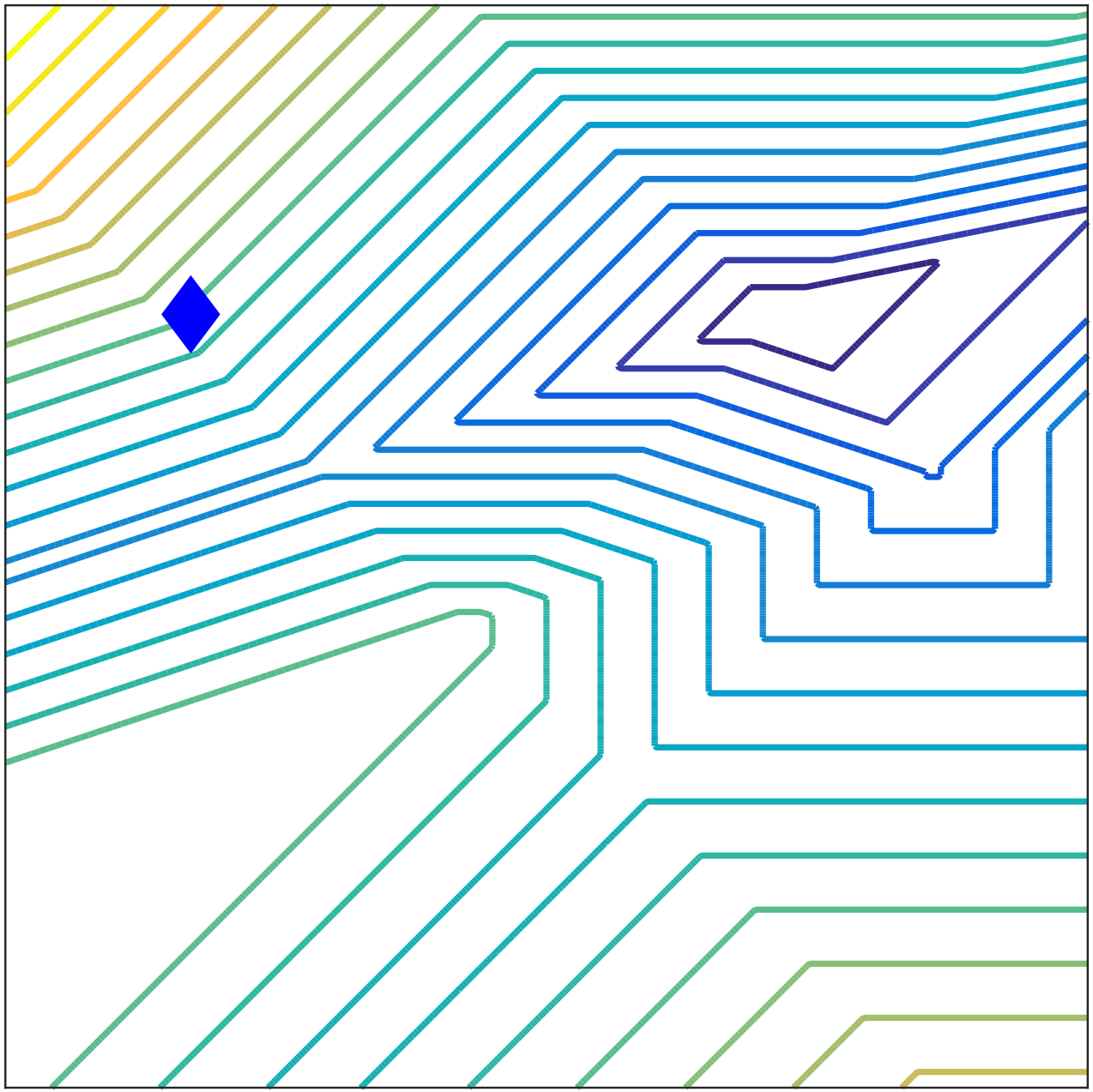

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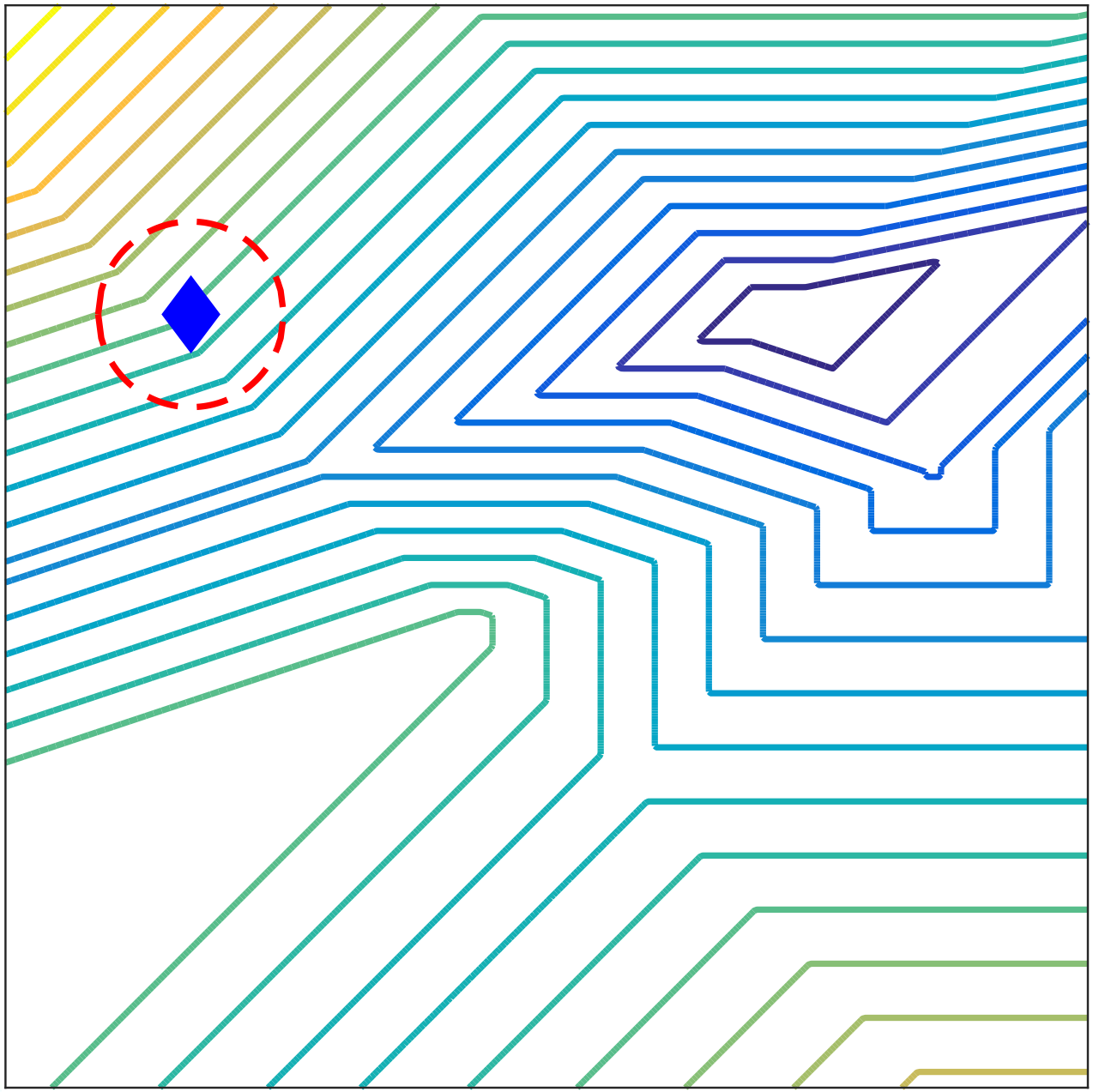

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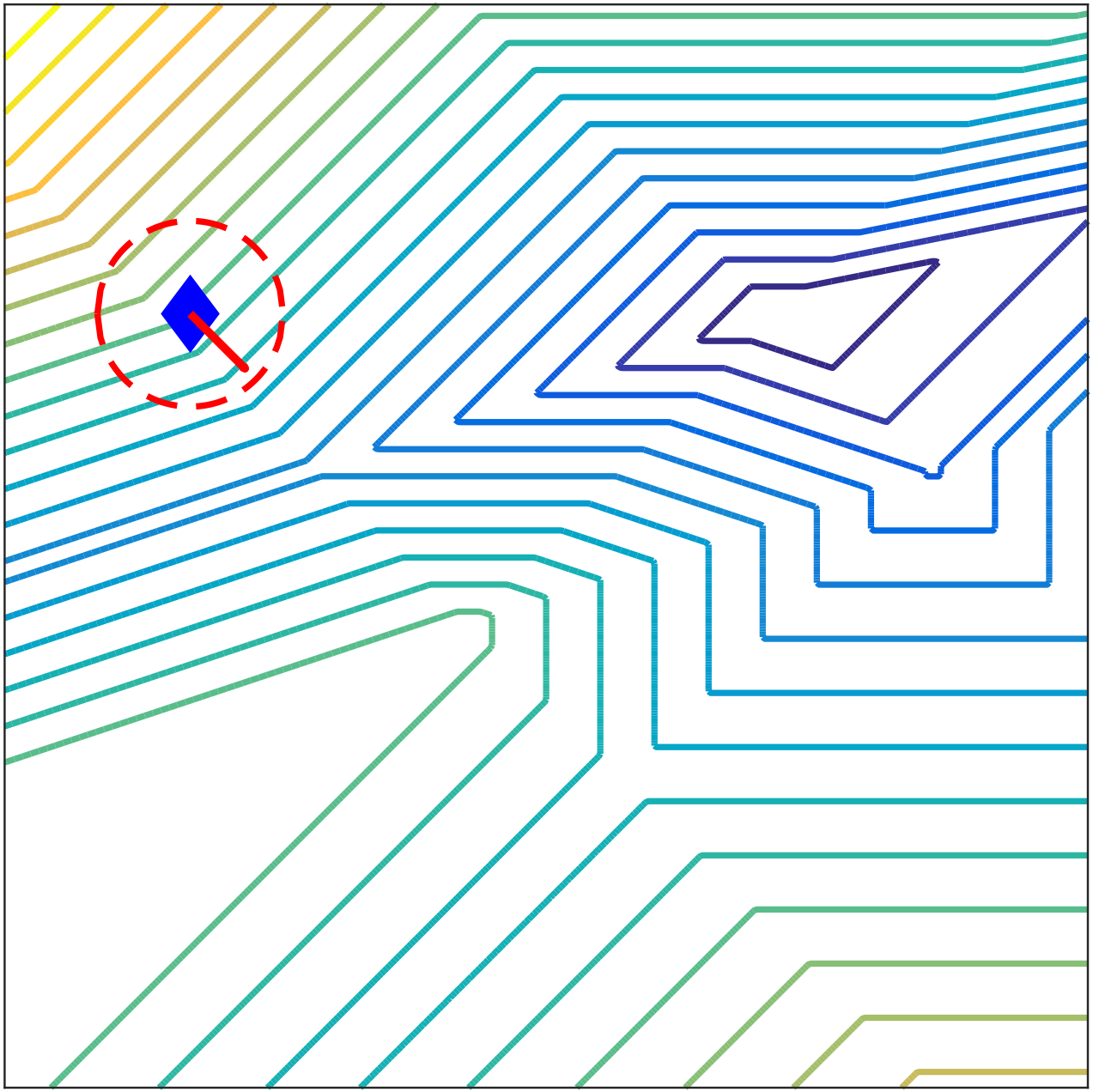

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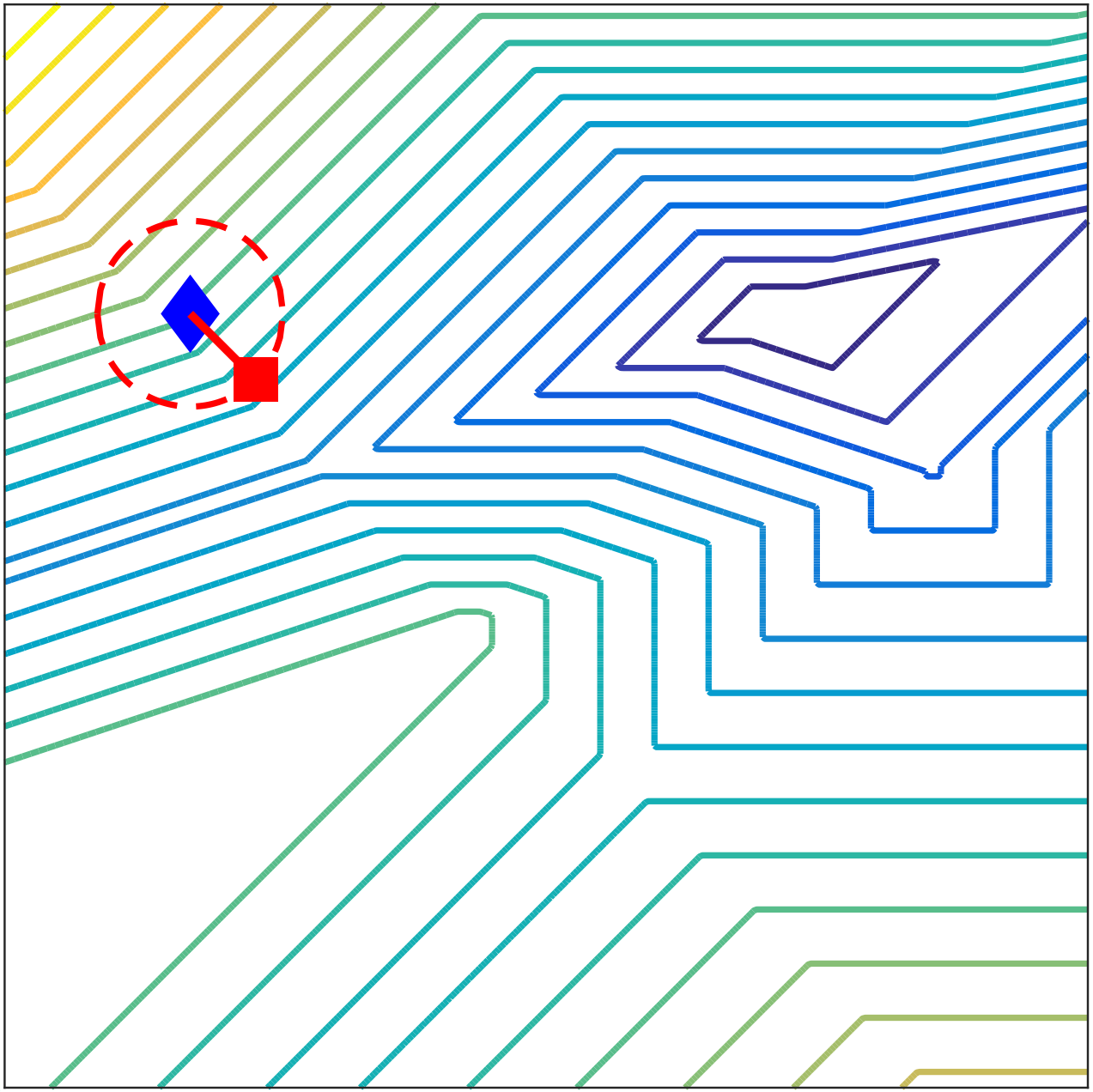

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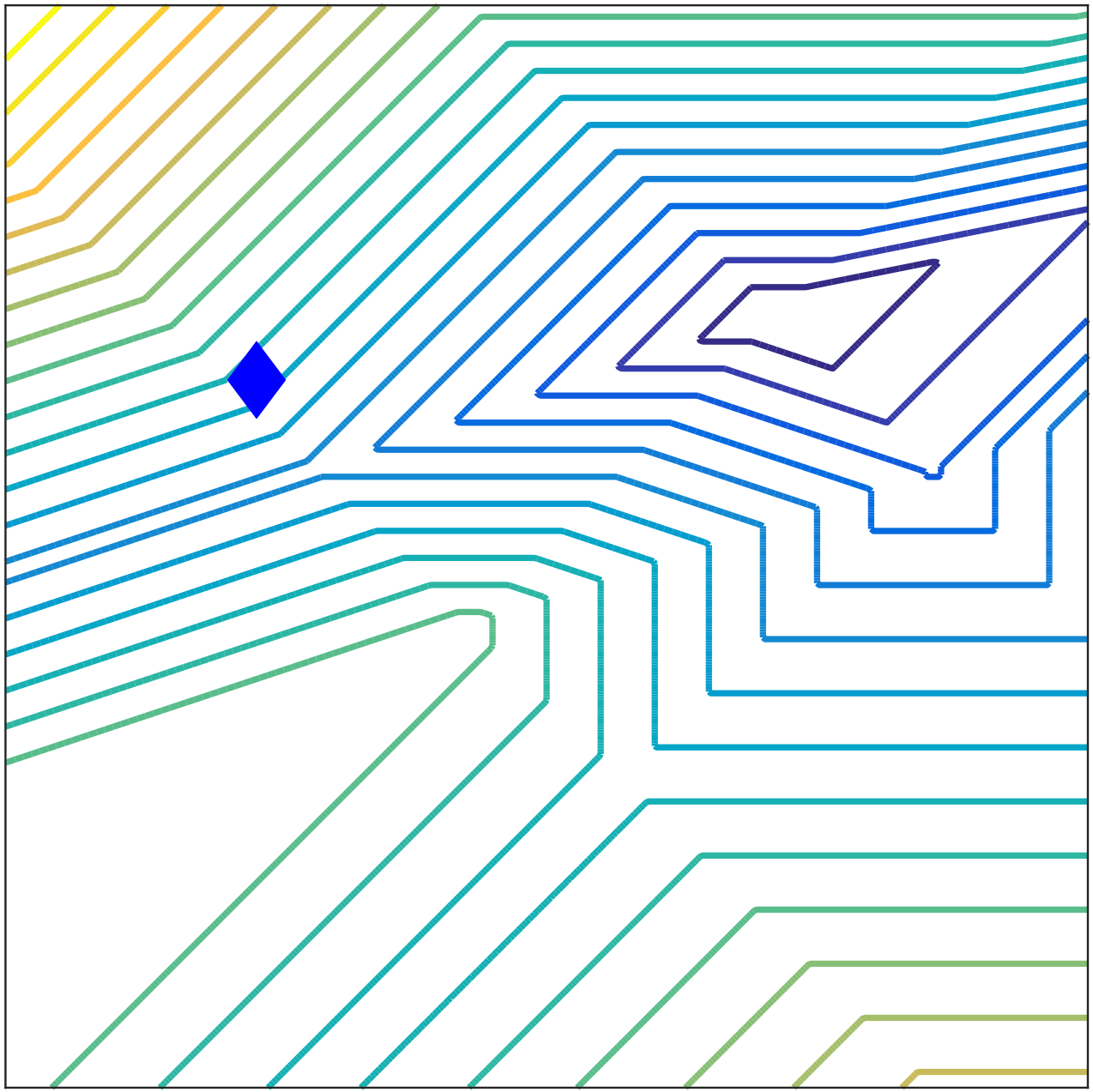

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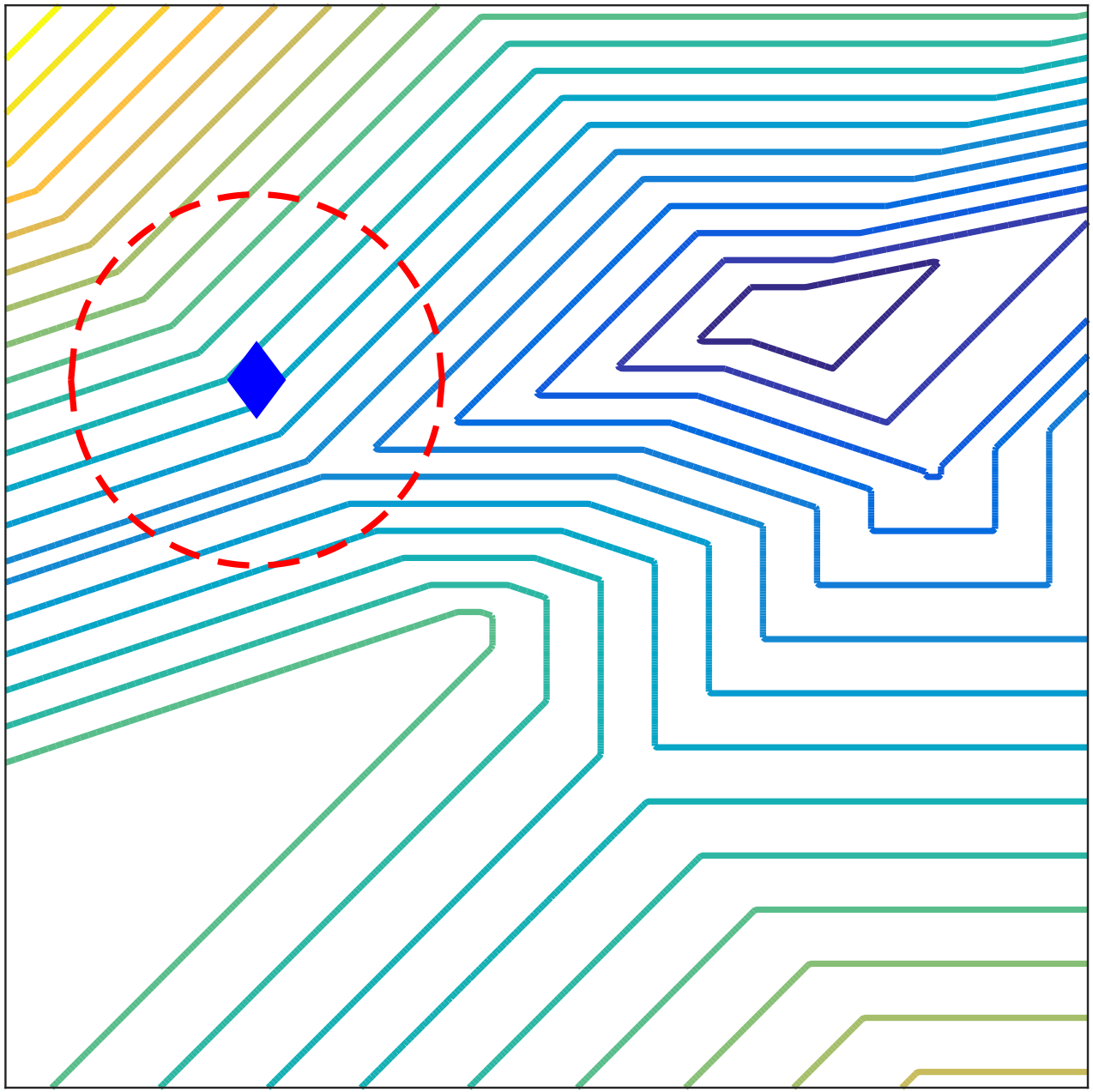

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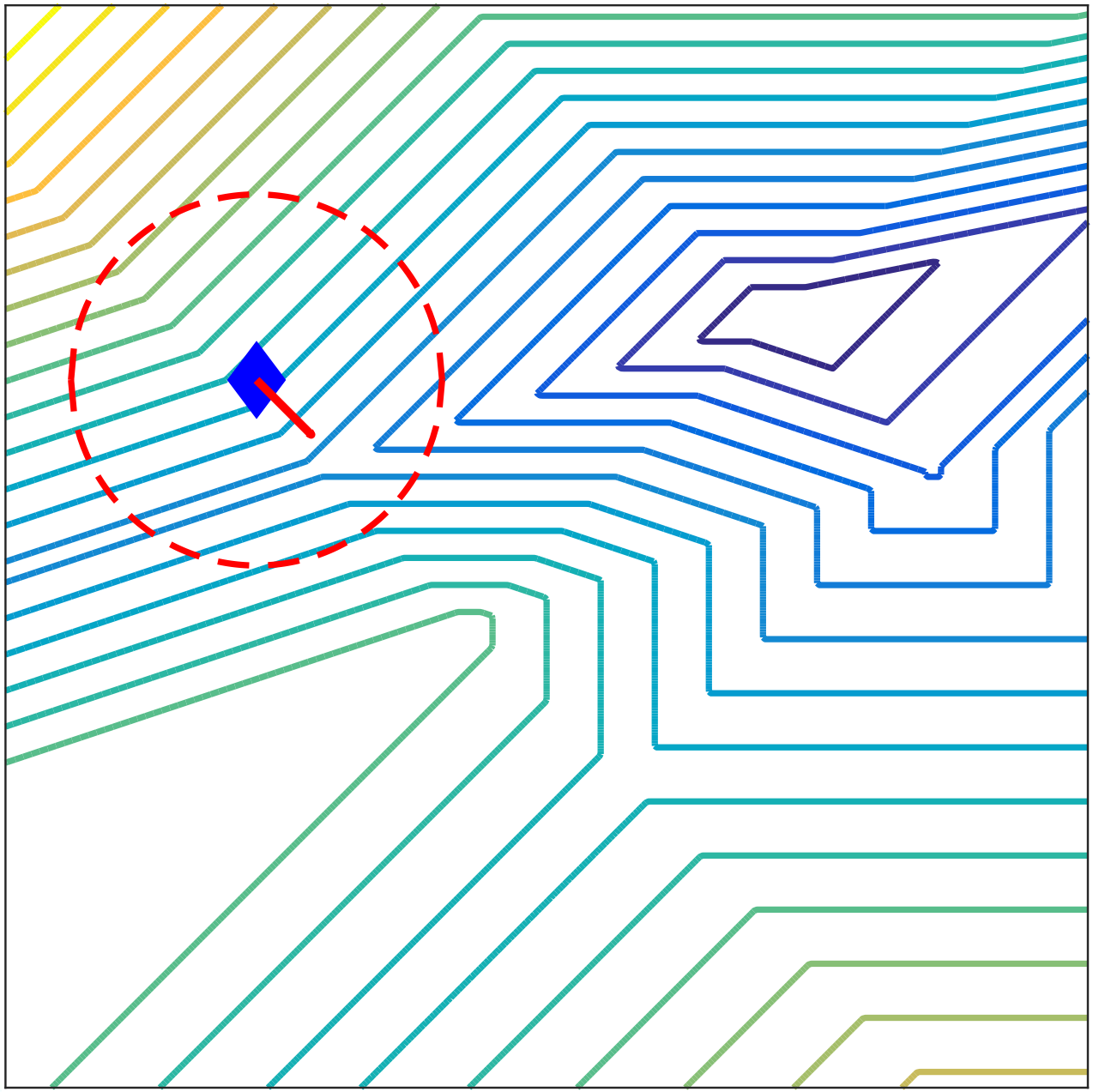

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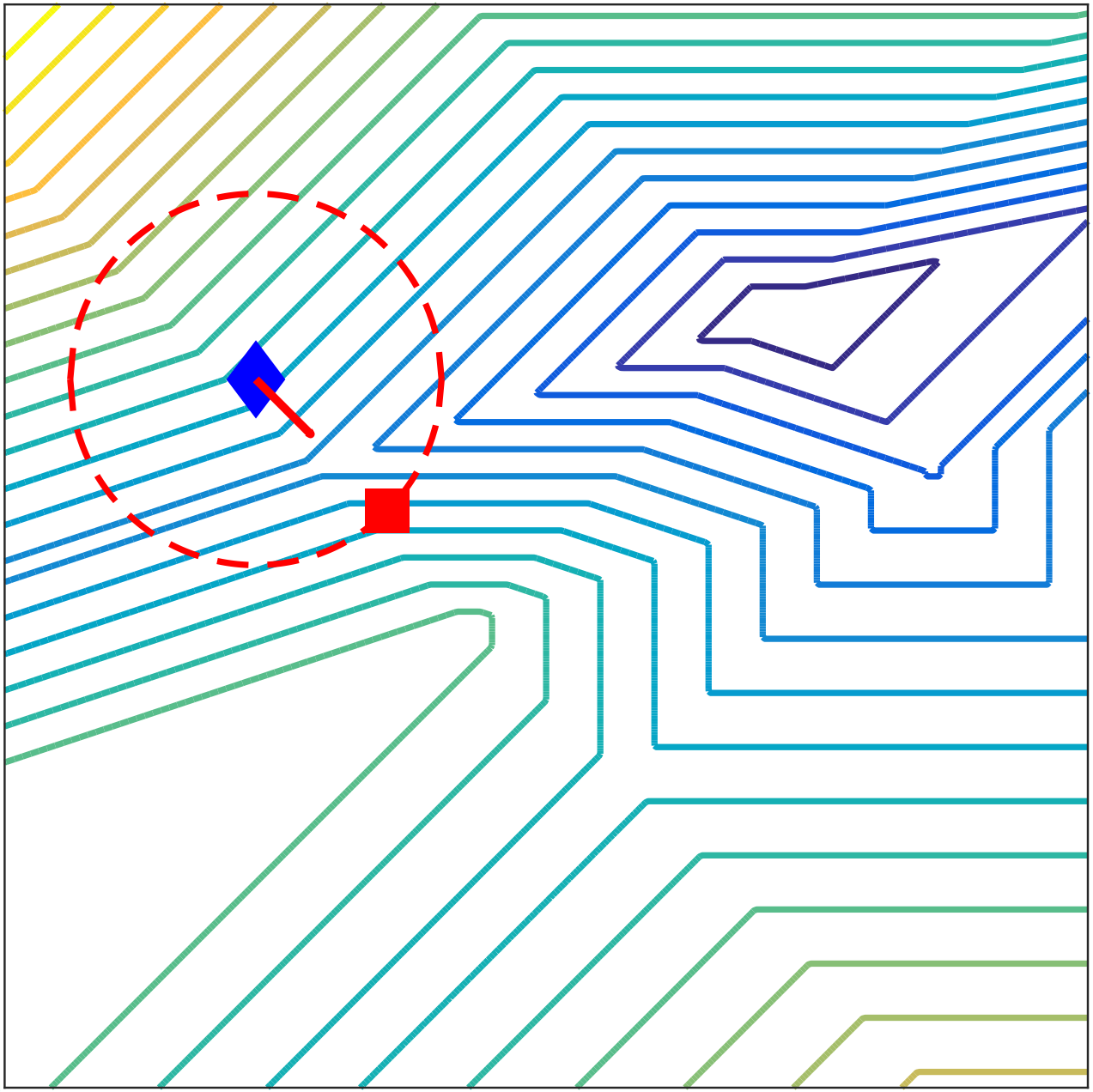

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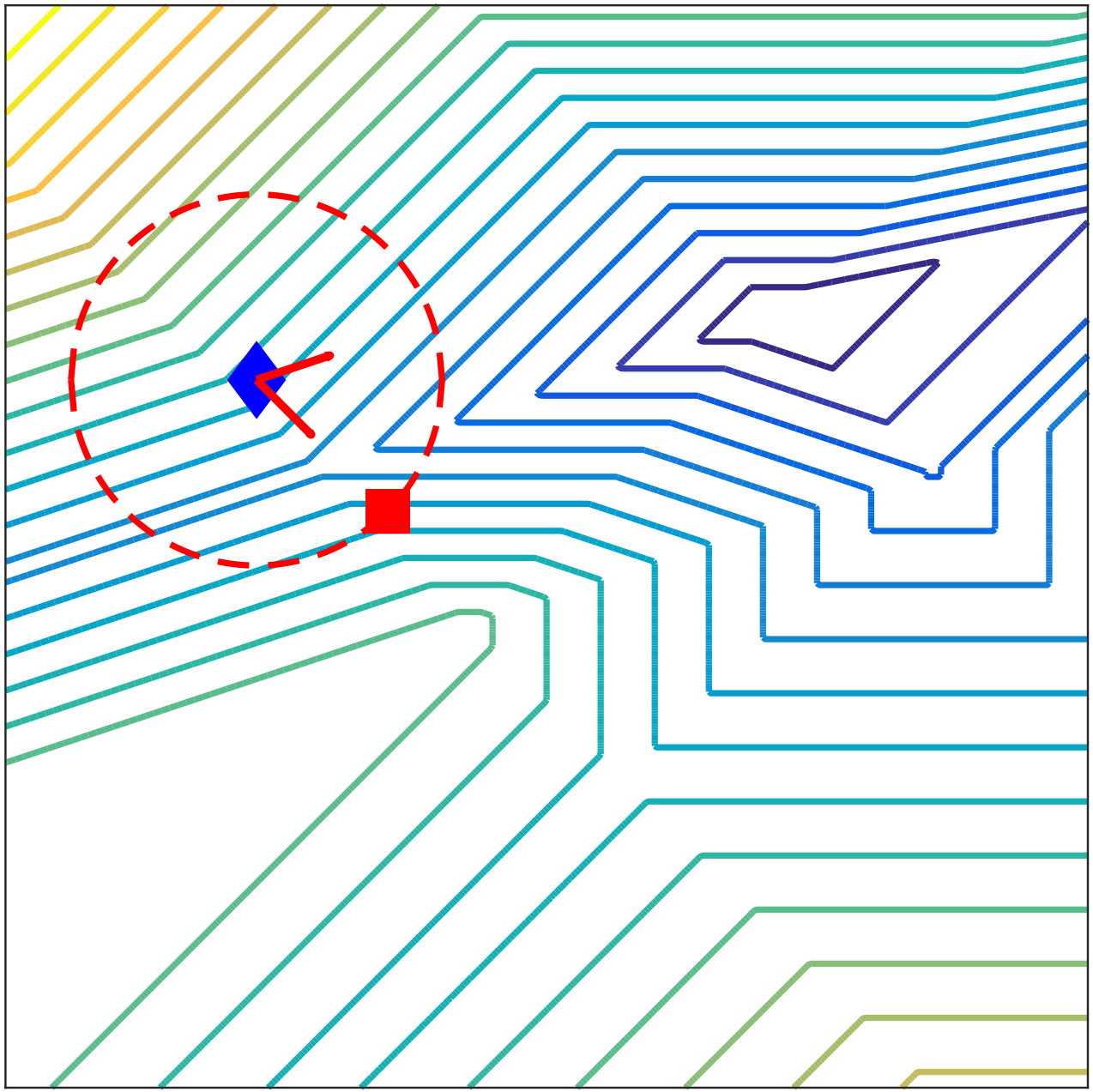

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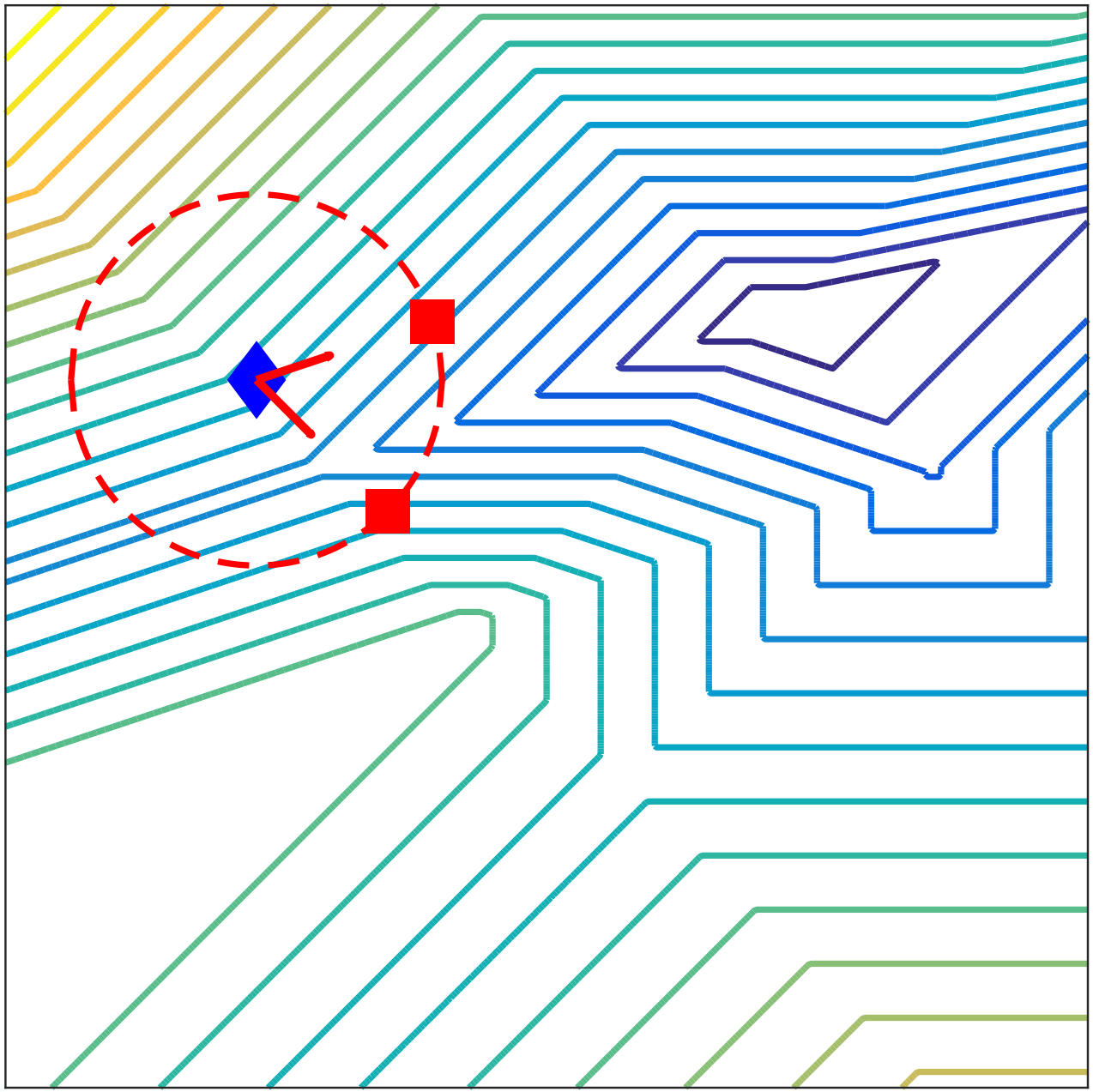

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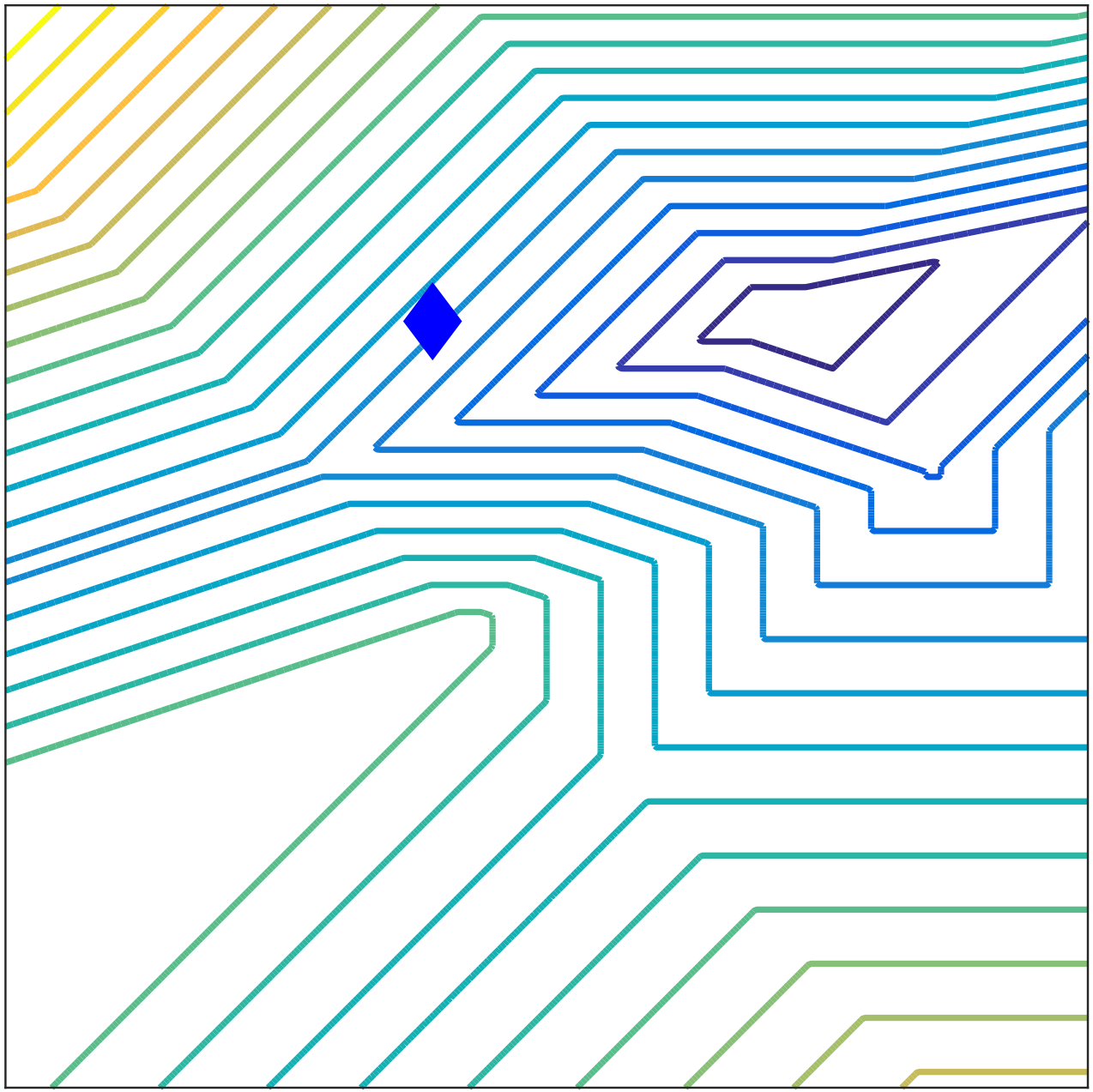

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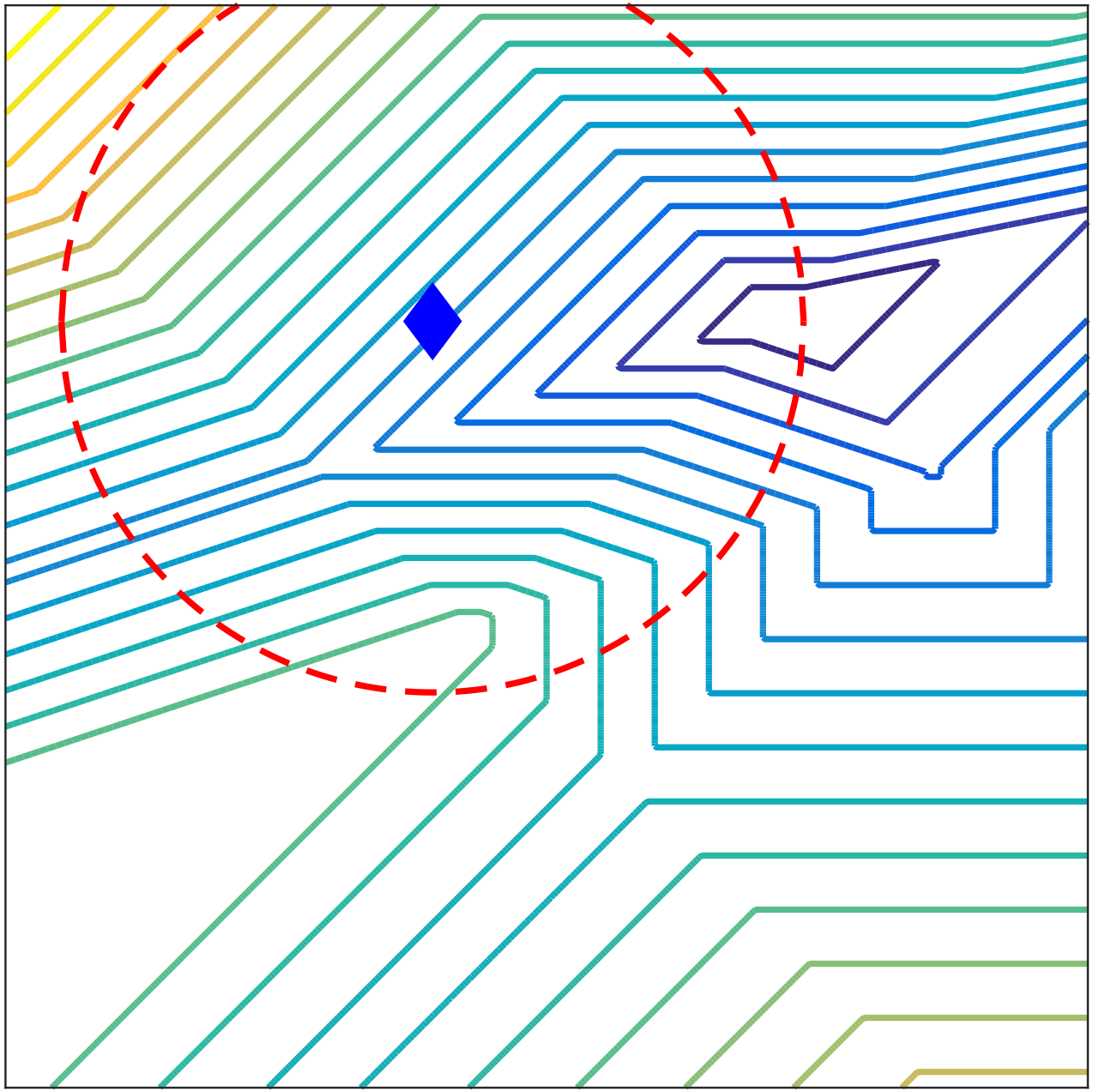

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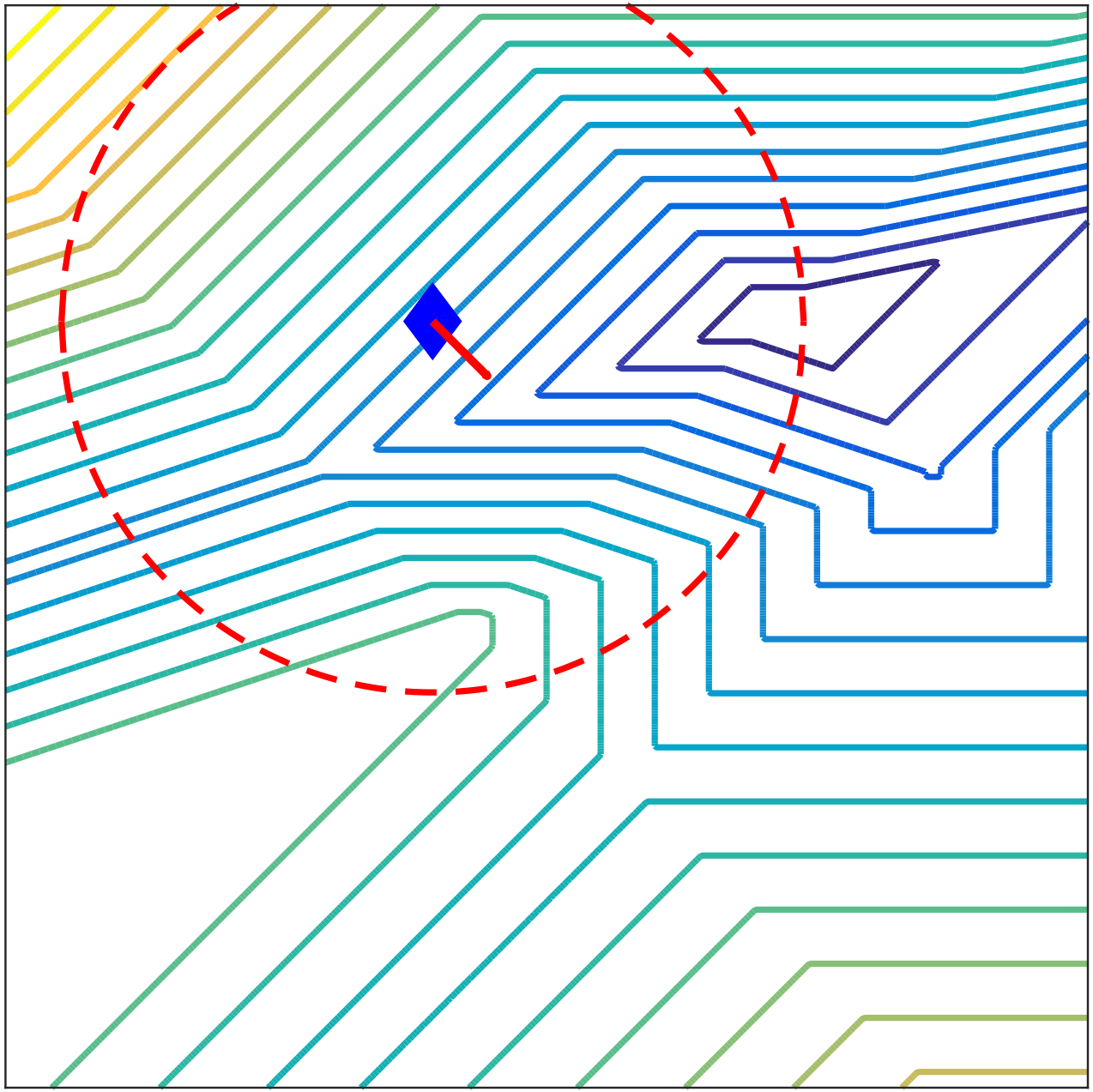

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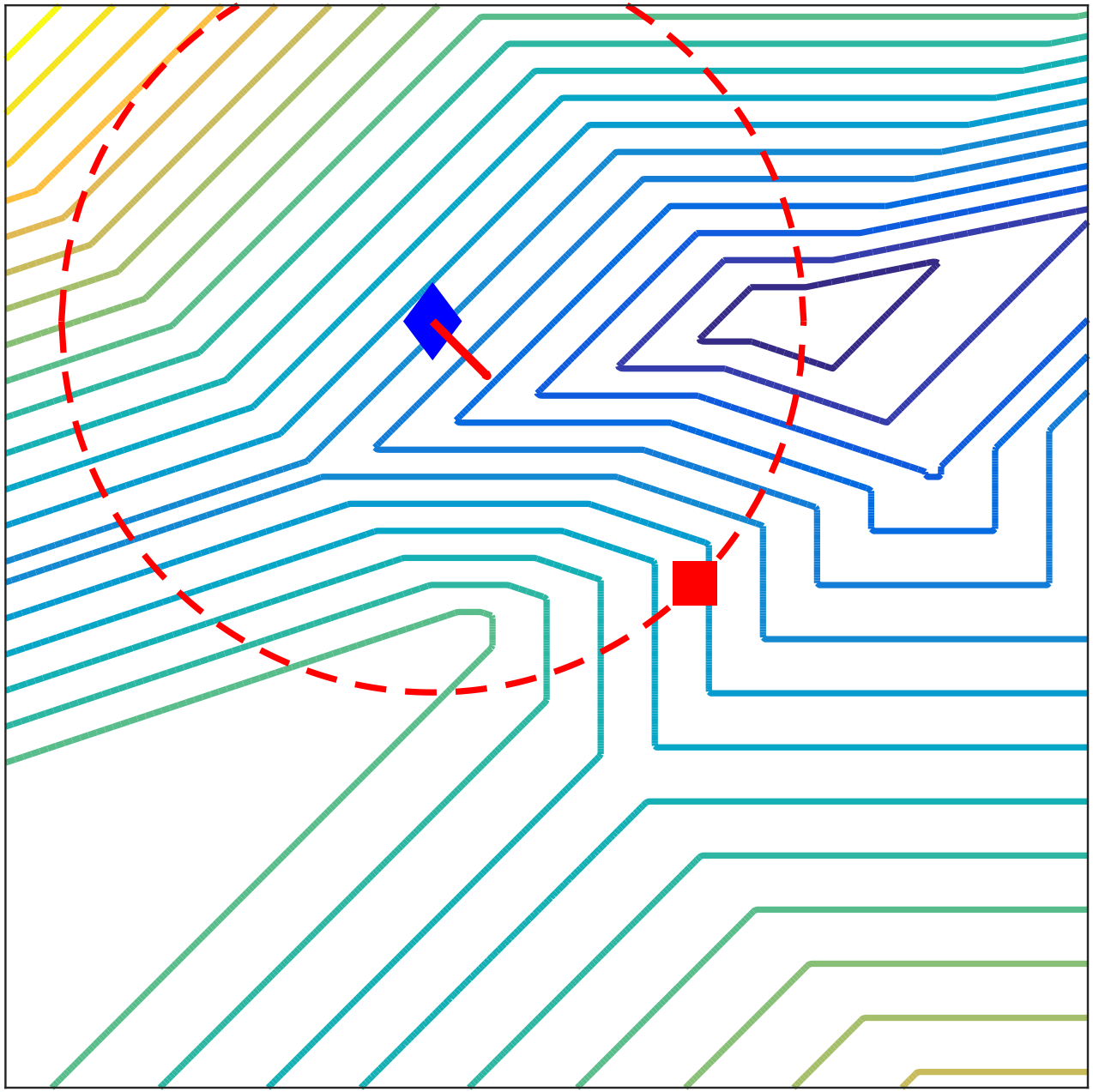

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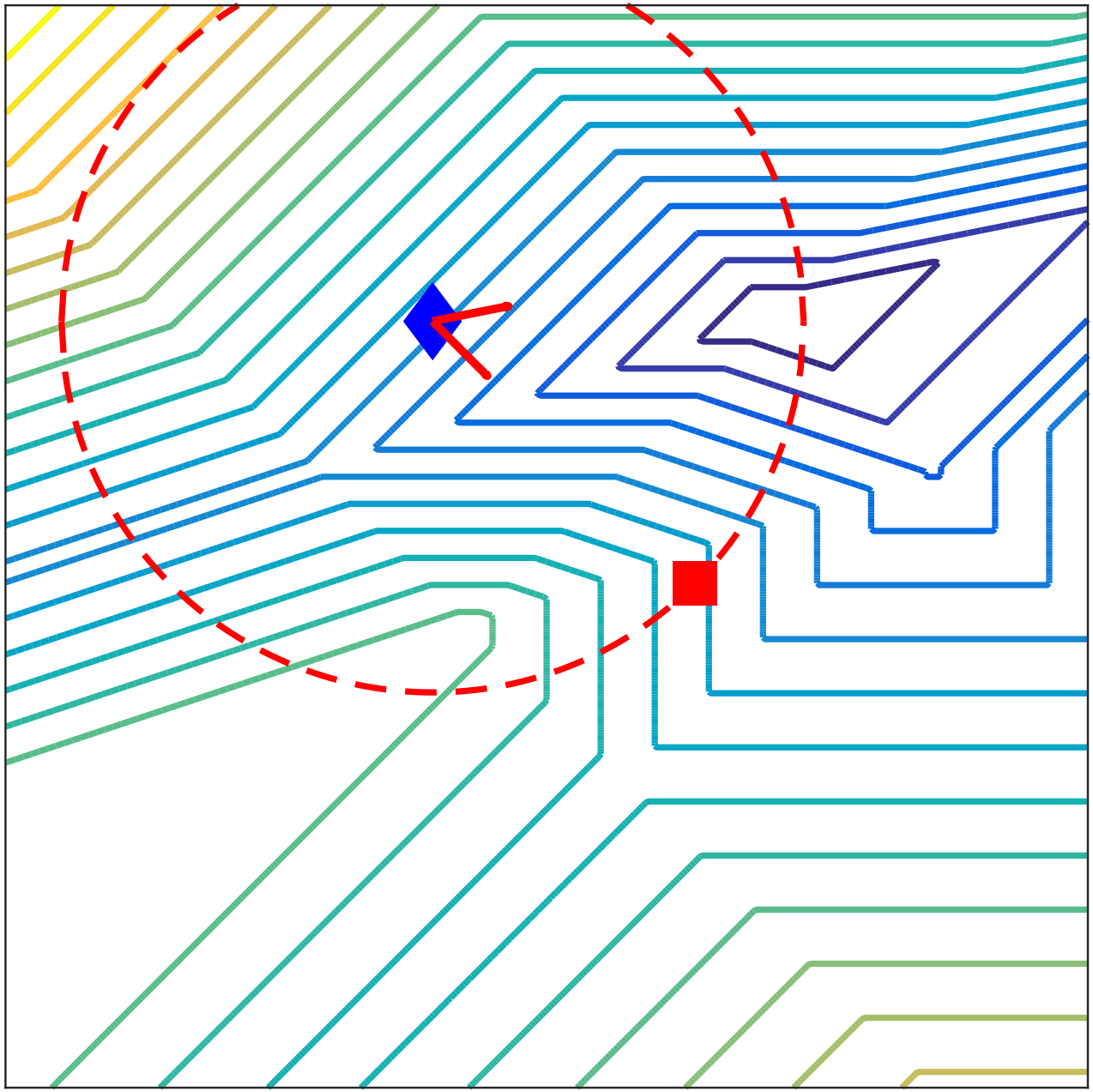

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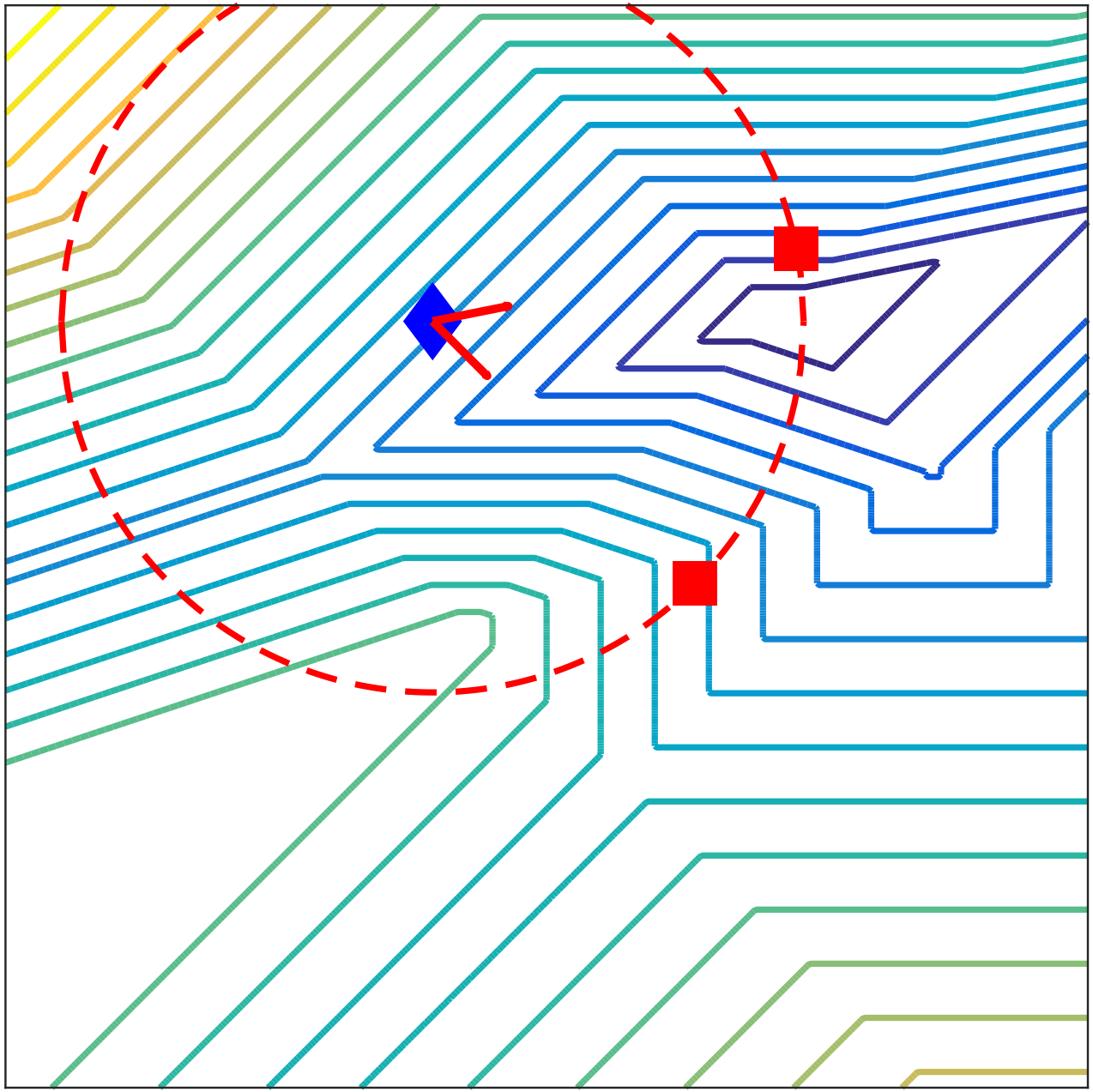

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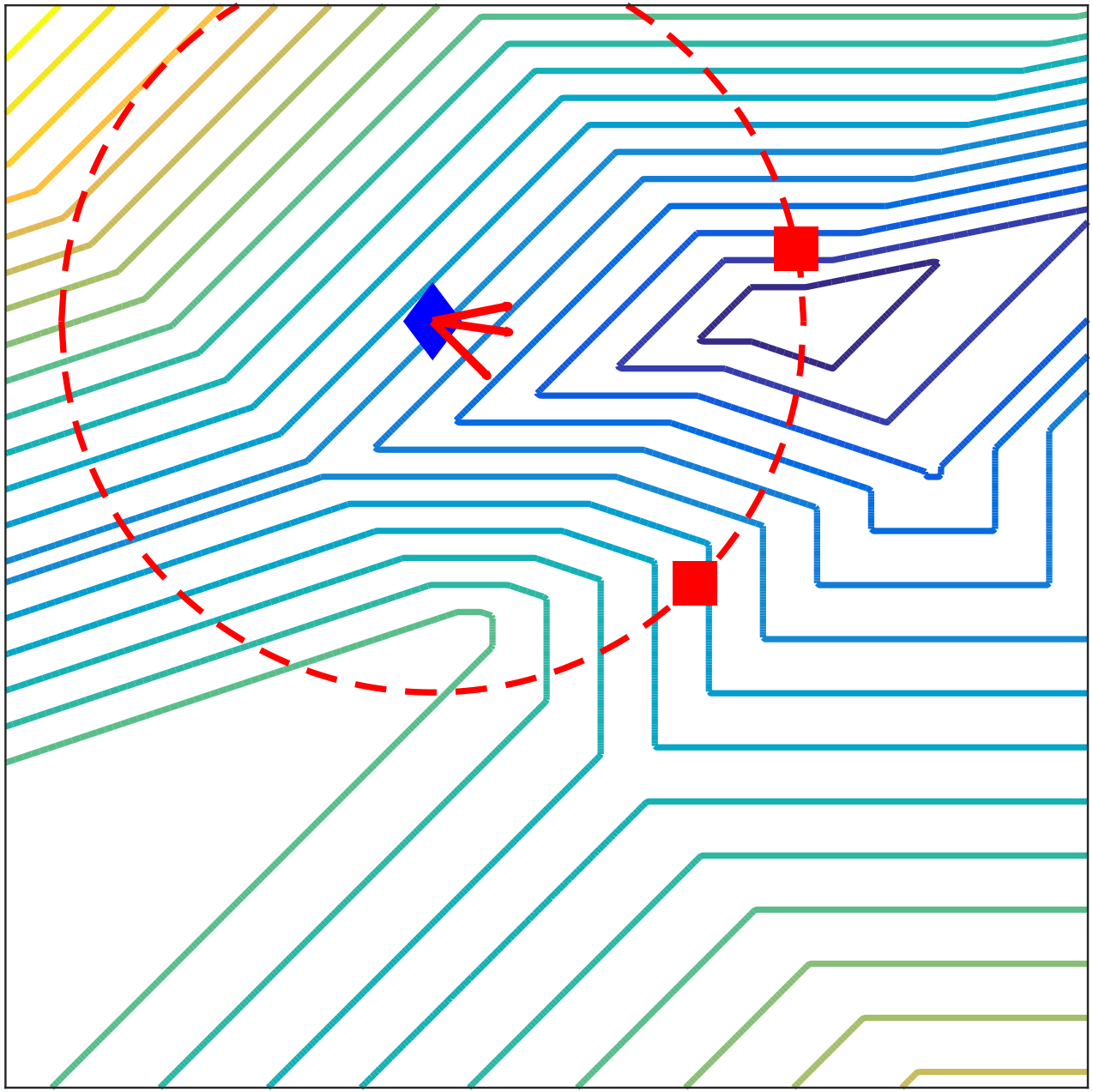

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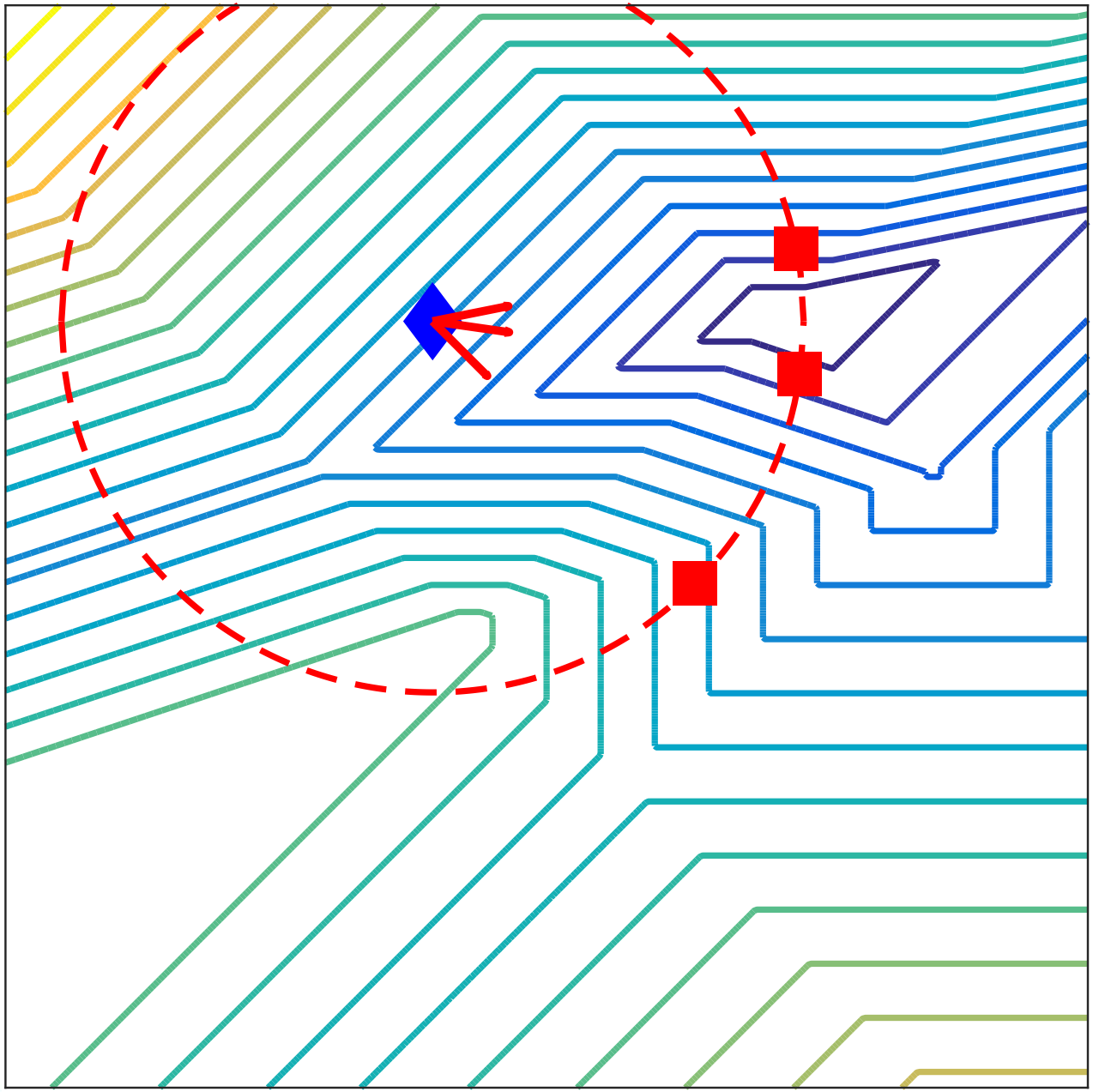

Smooth master model

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Define the smooth *master model* $m_k^f : \mathbb{R}^n \to \mathbb{R}$ (with gradient g^k) and obtain step by (approximately) solving

$$\begin{aligned} &\underset{s}{\text{minimize}}\ m_k^f(x^k + s) \\ &\text{subject to: } s \in \mathcal{B}(0, \Delta_k) \end{aligned}$$

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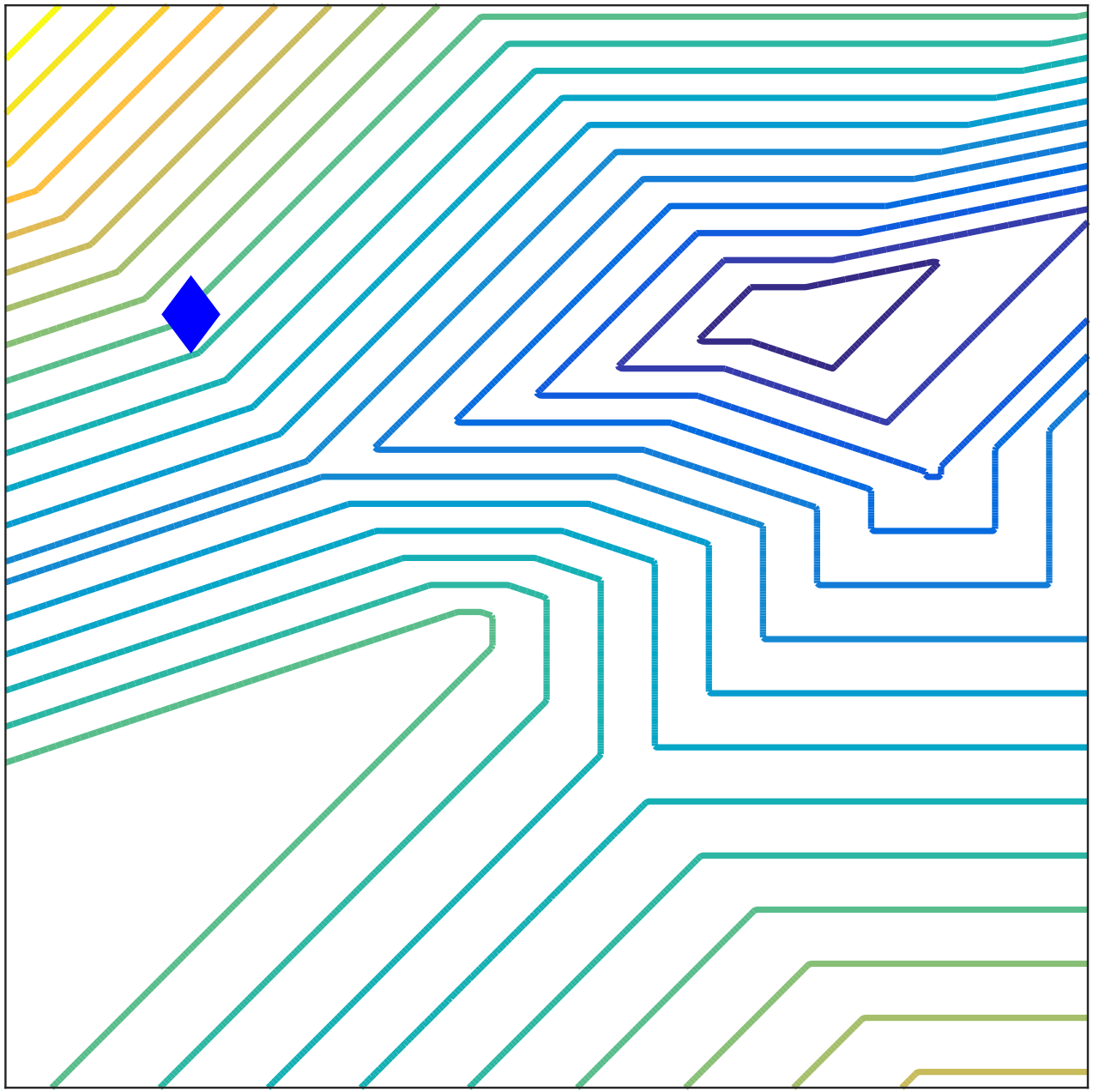

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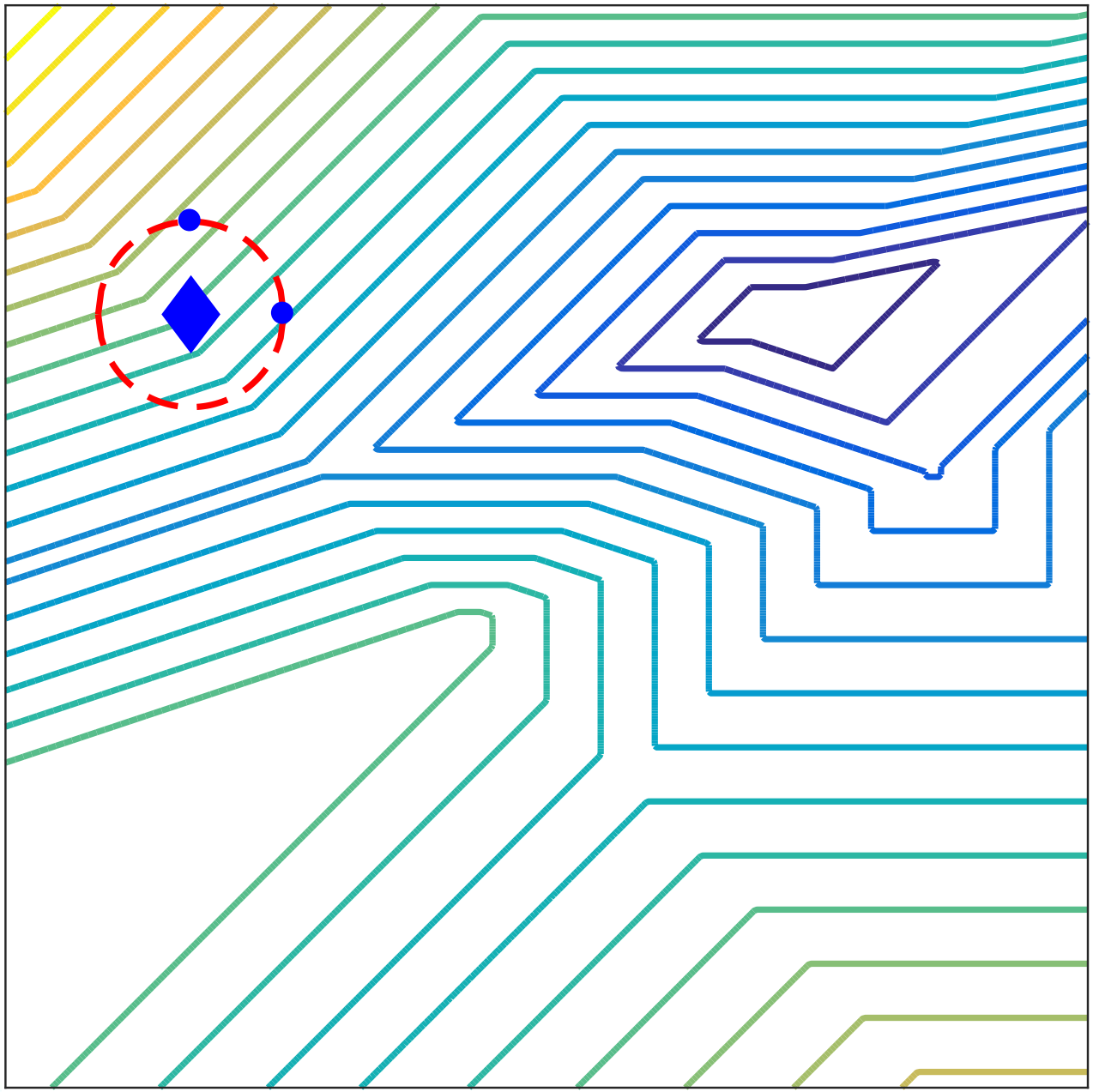

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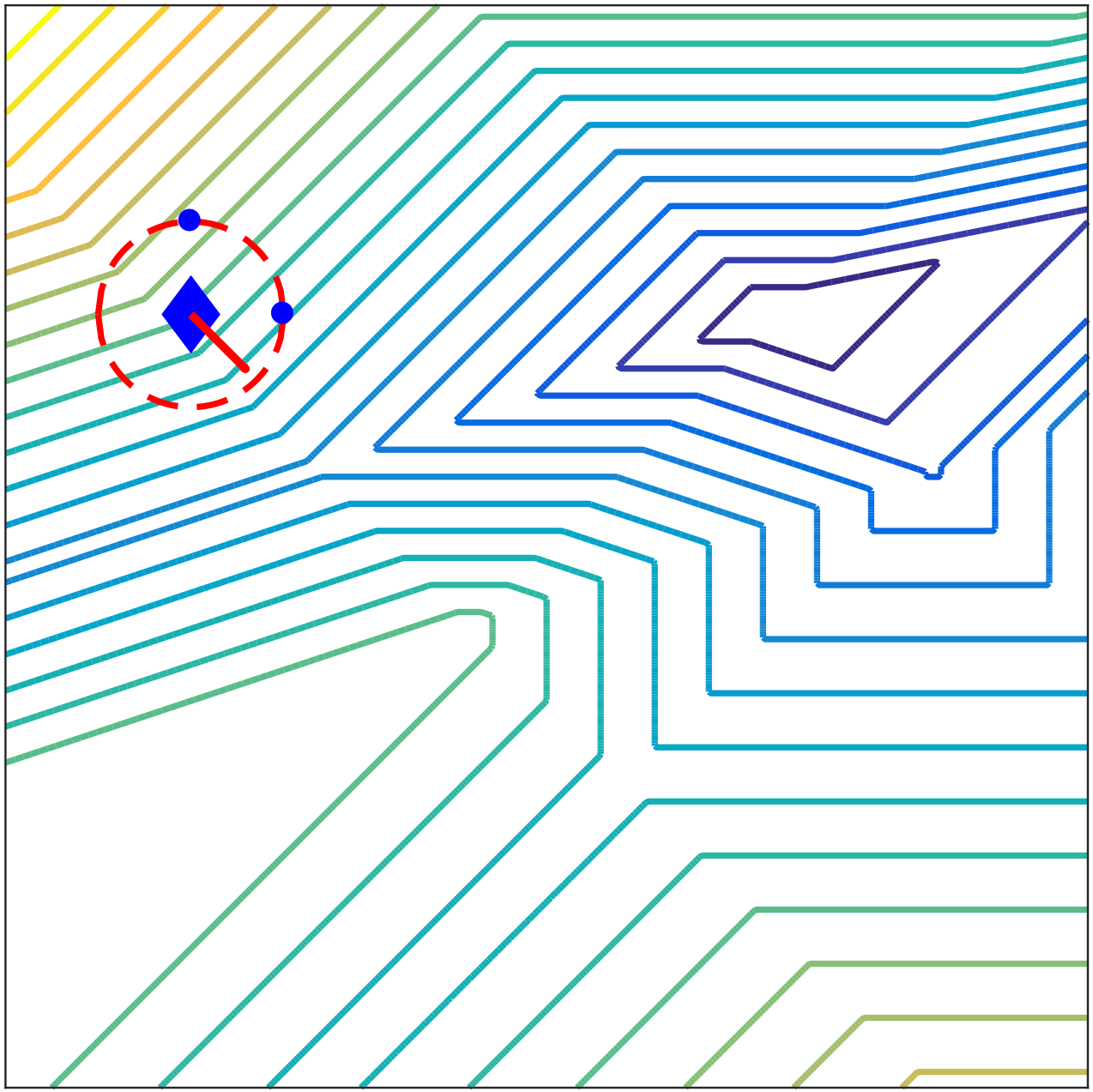

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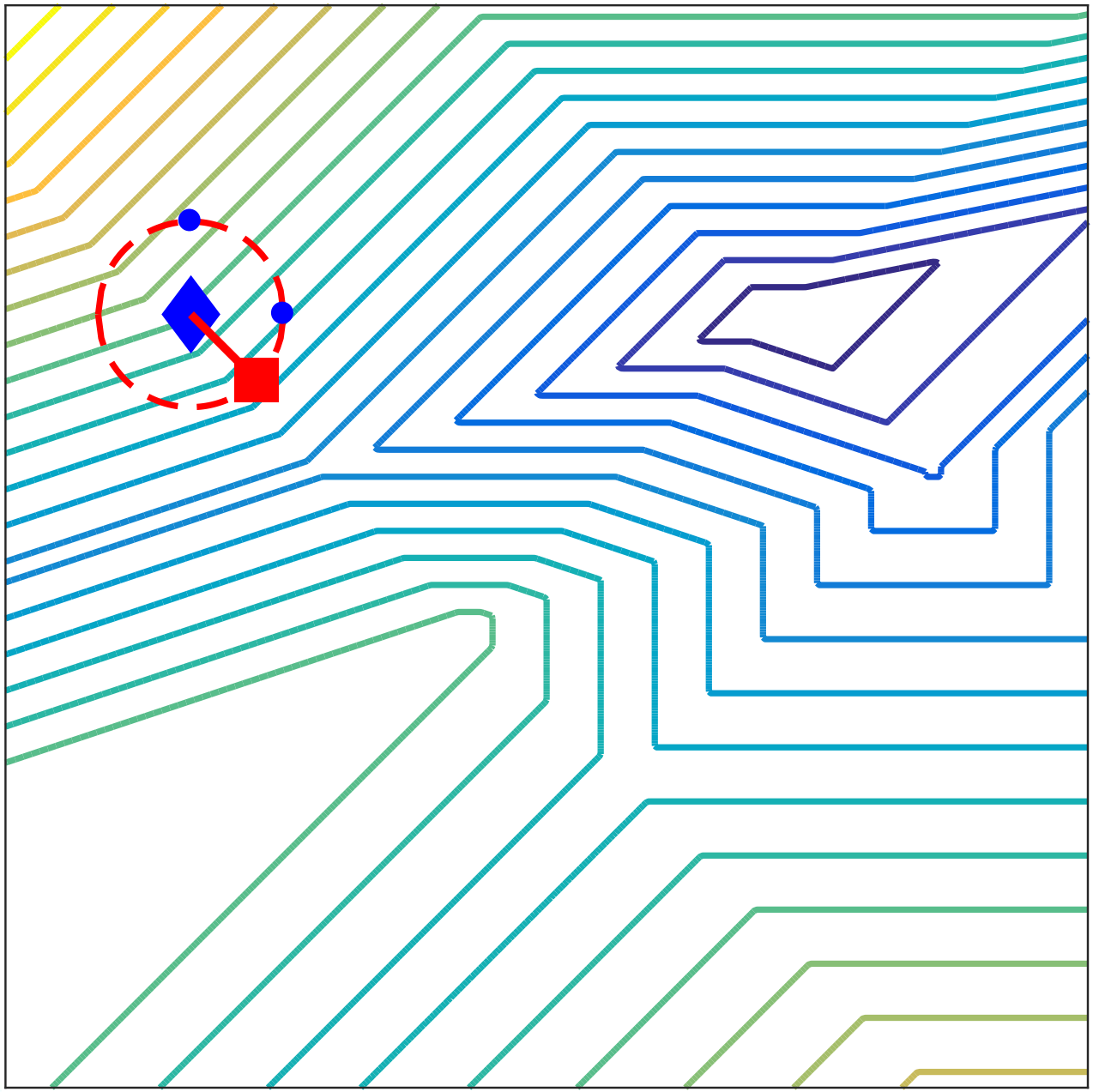

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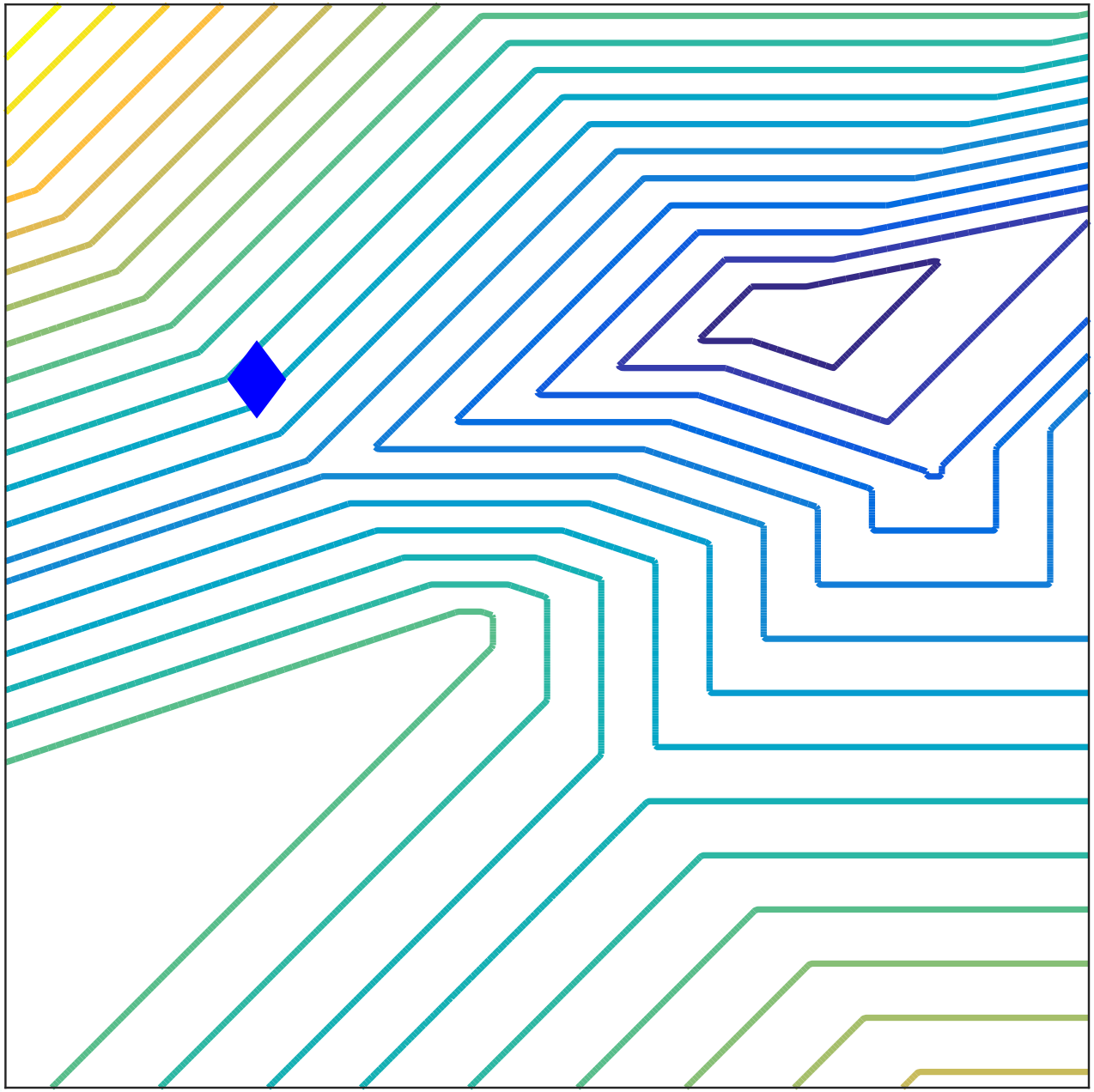

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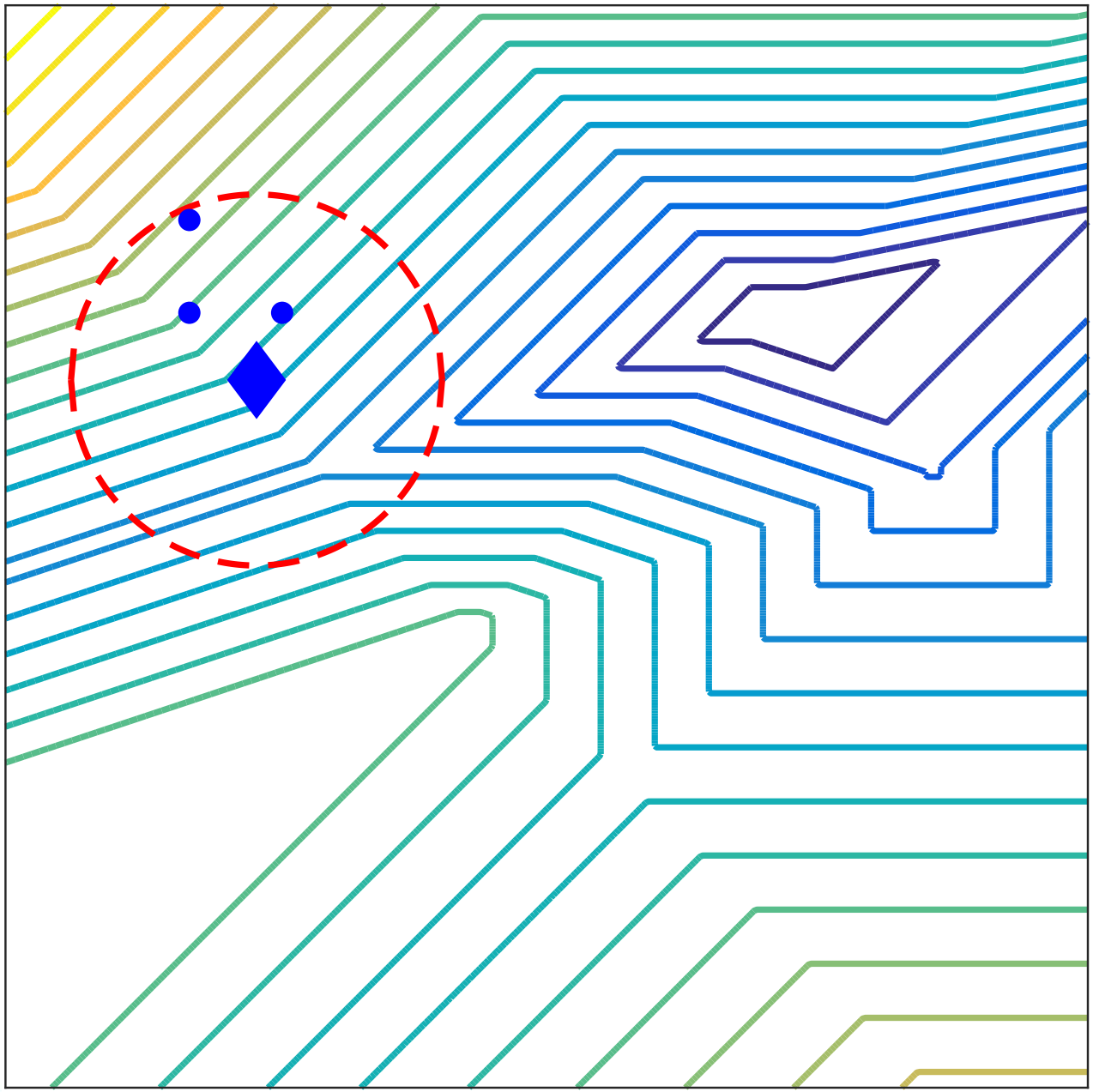

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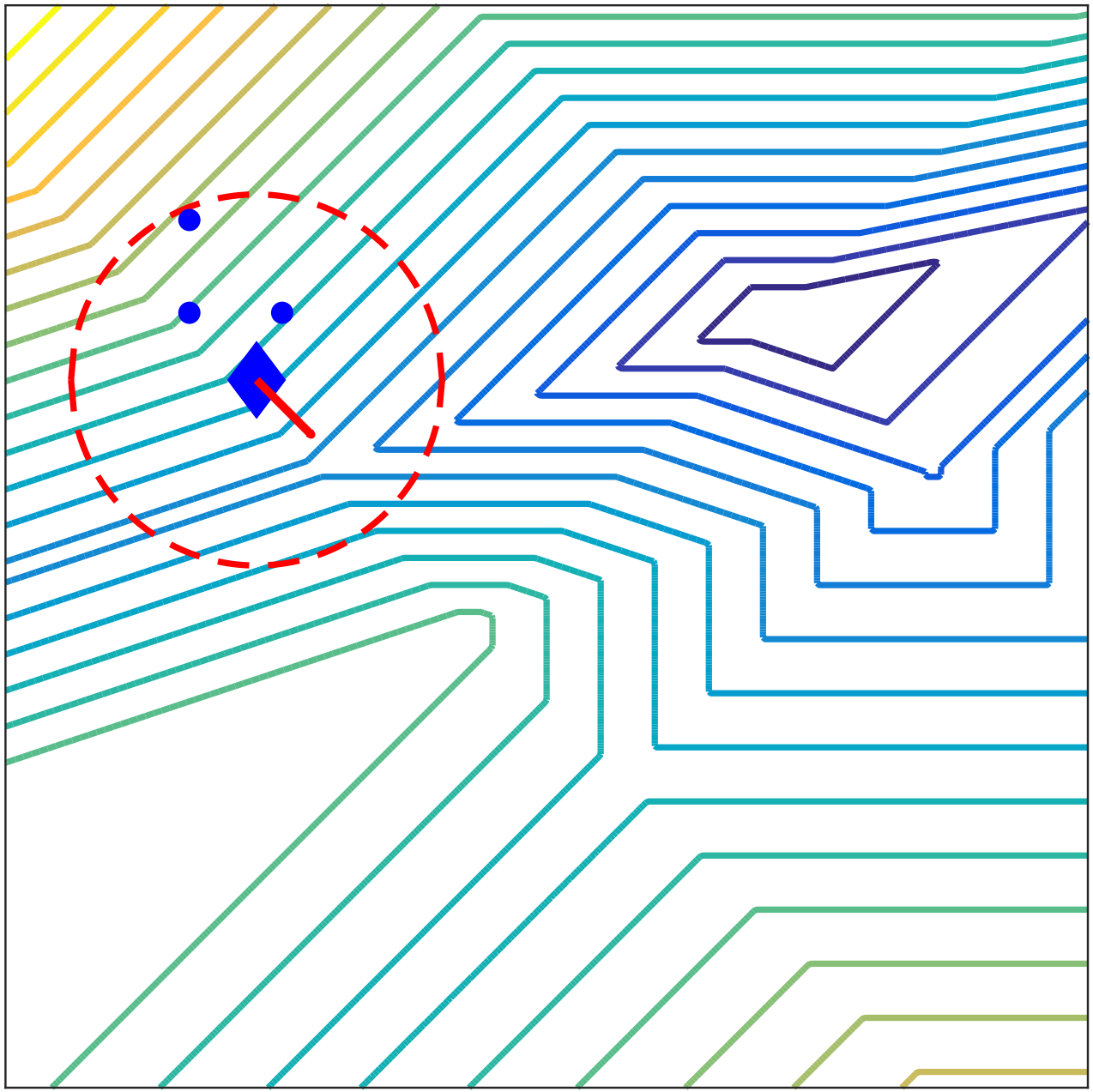

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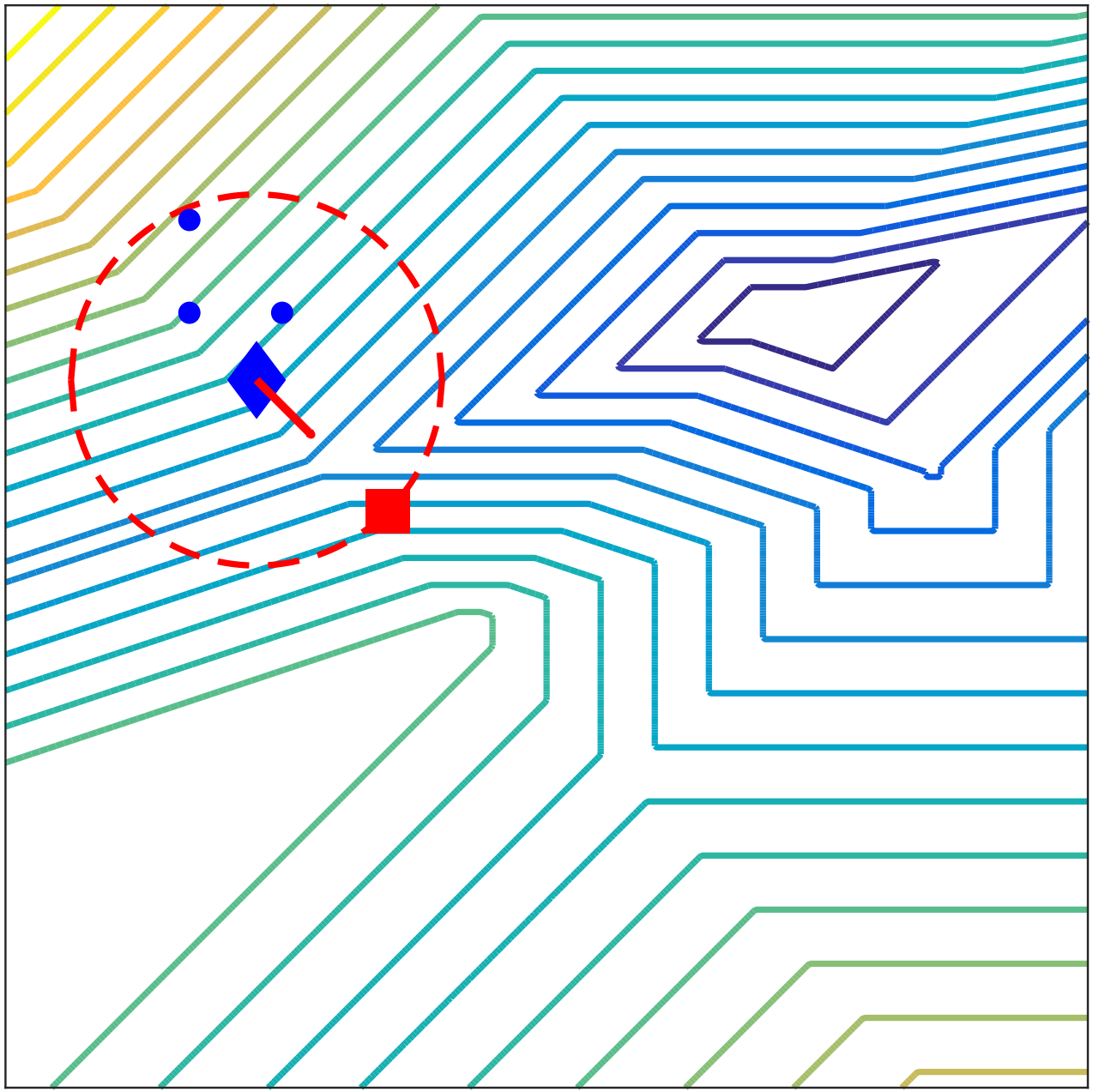

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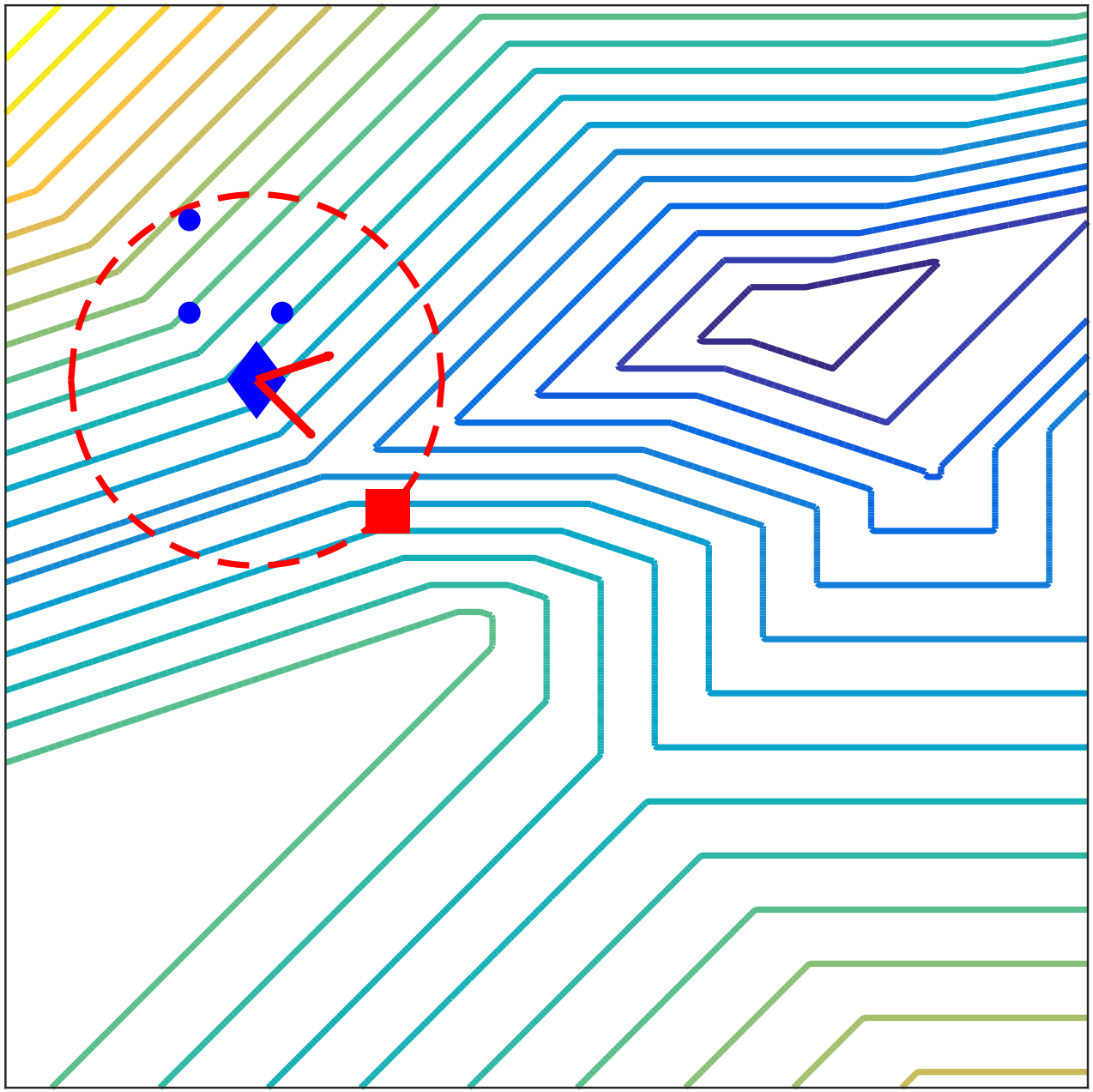

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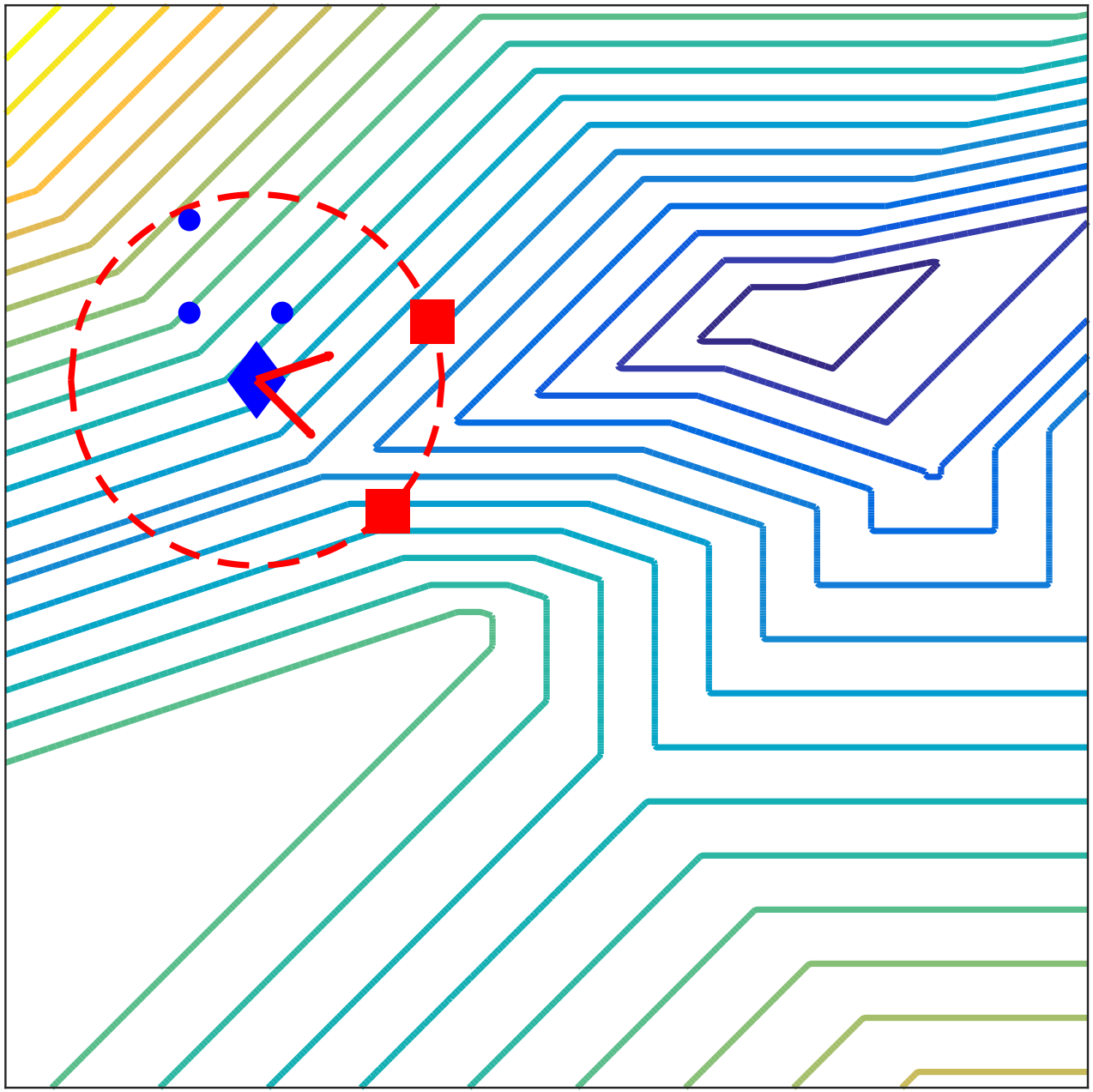

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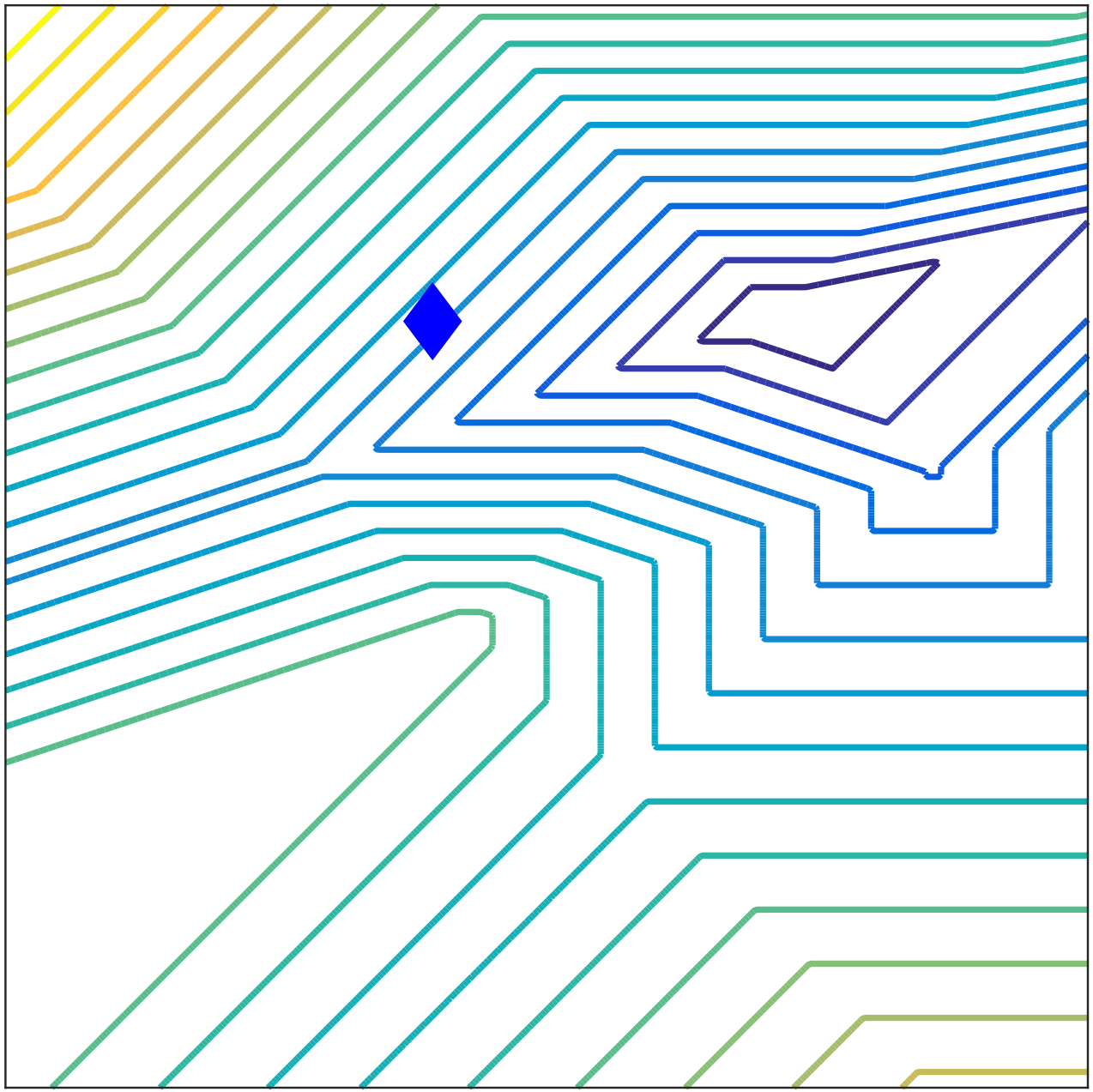

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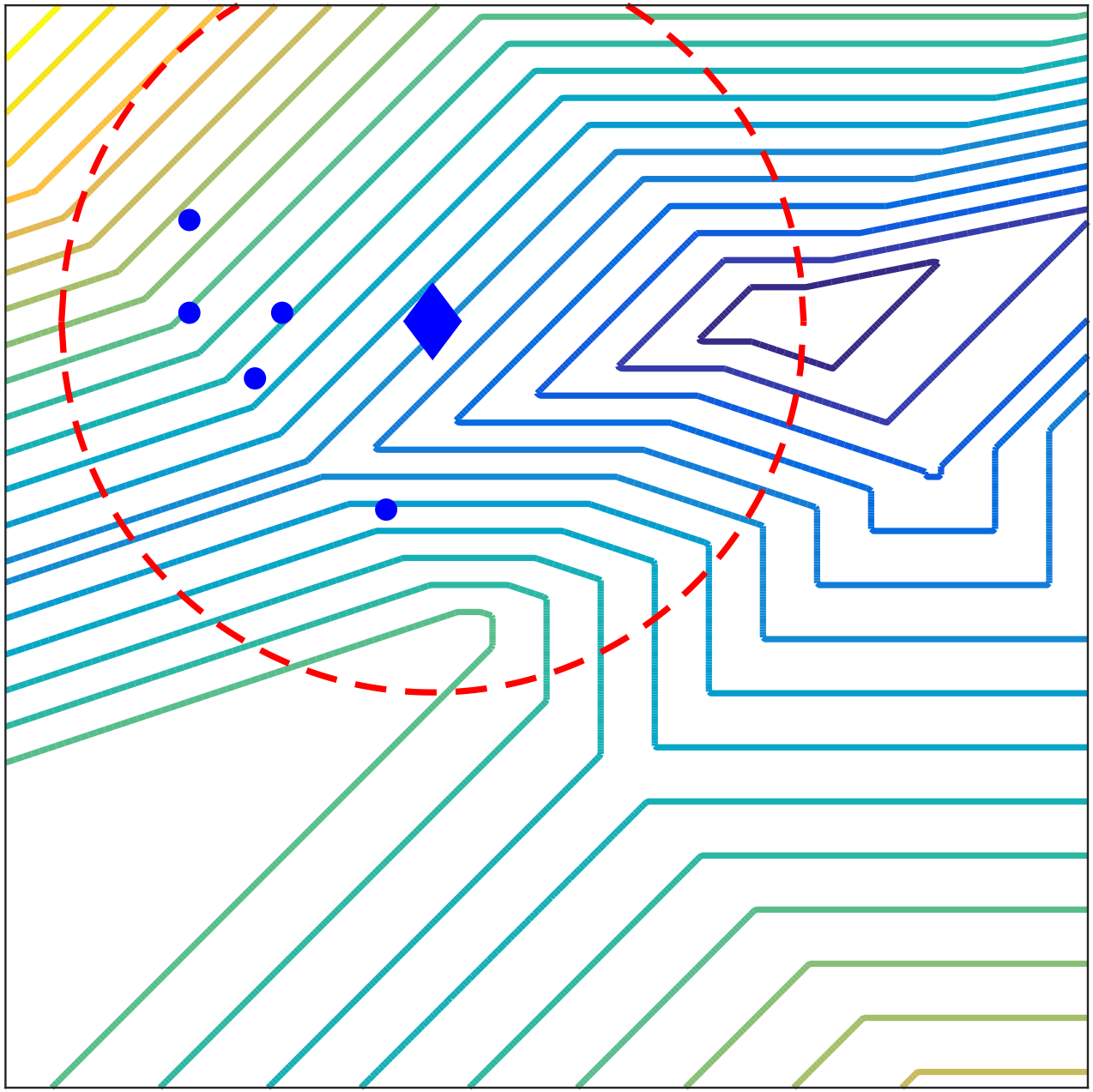

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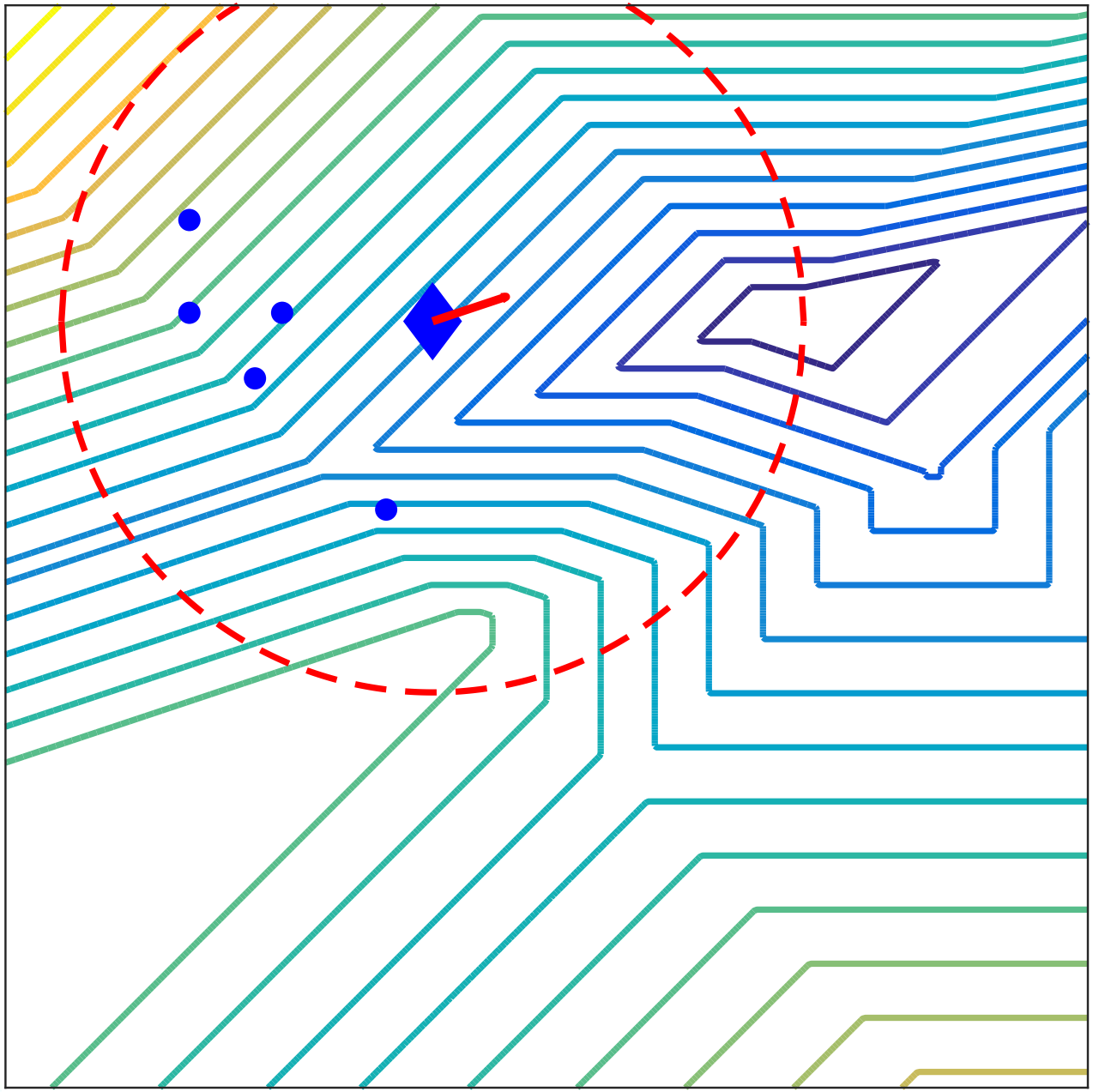

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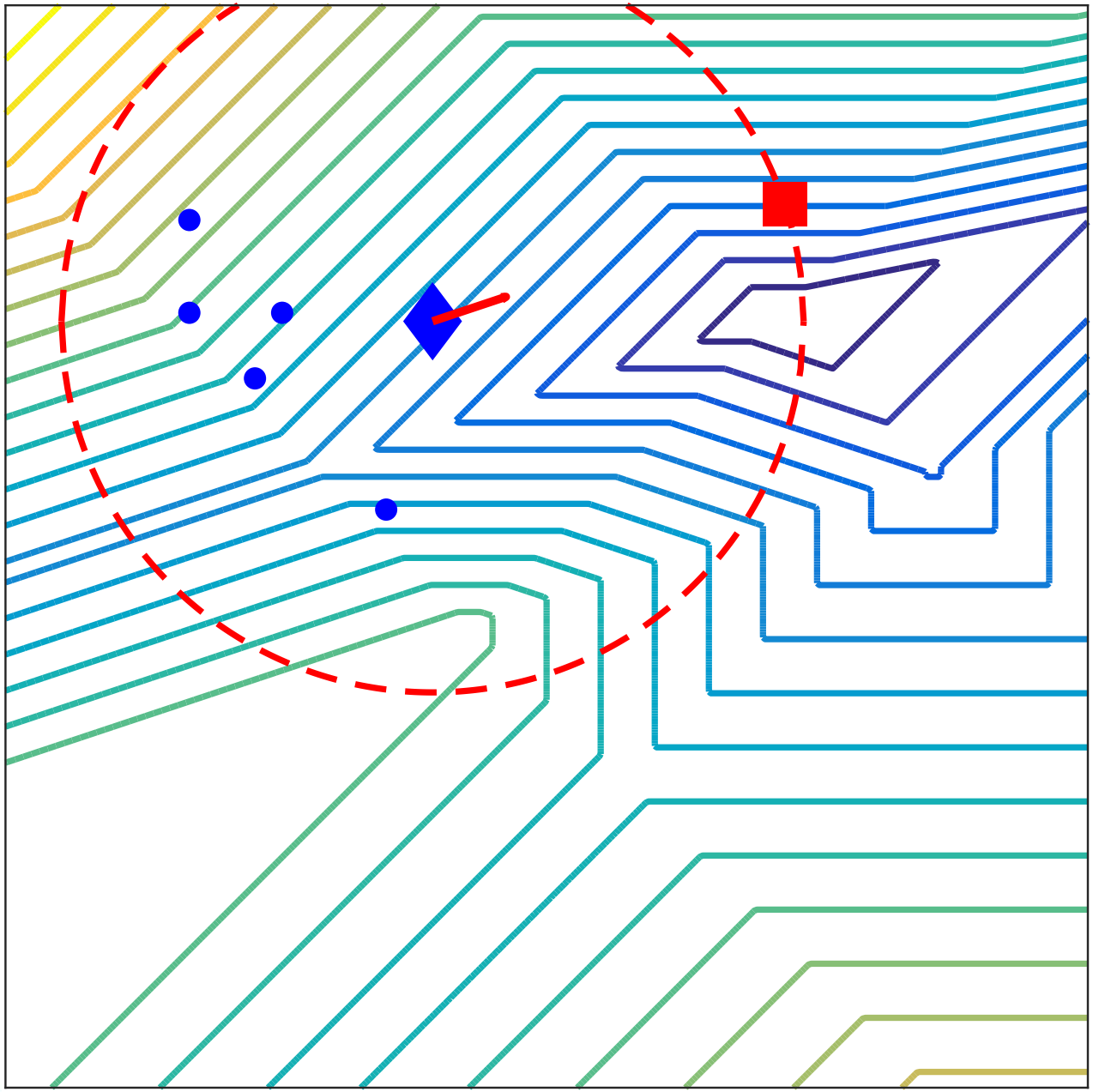

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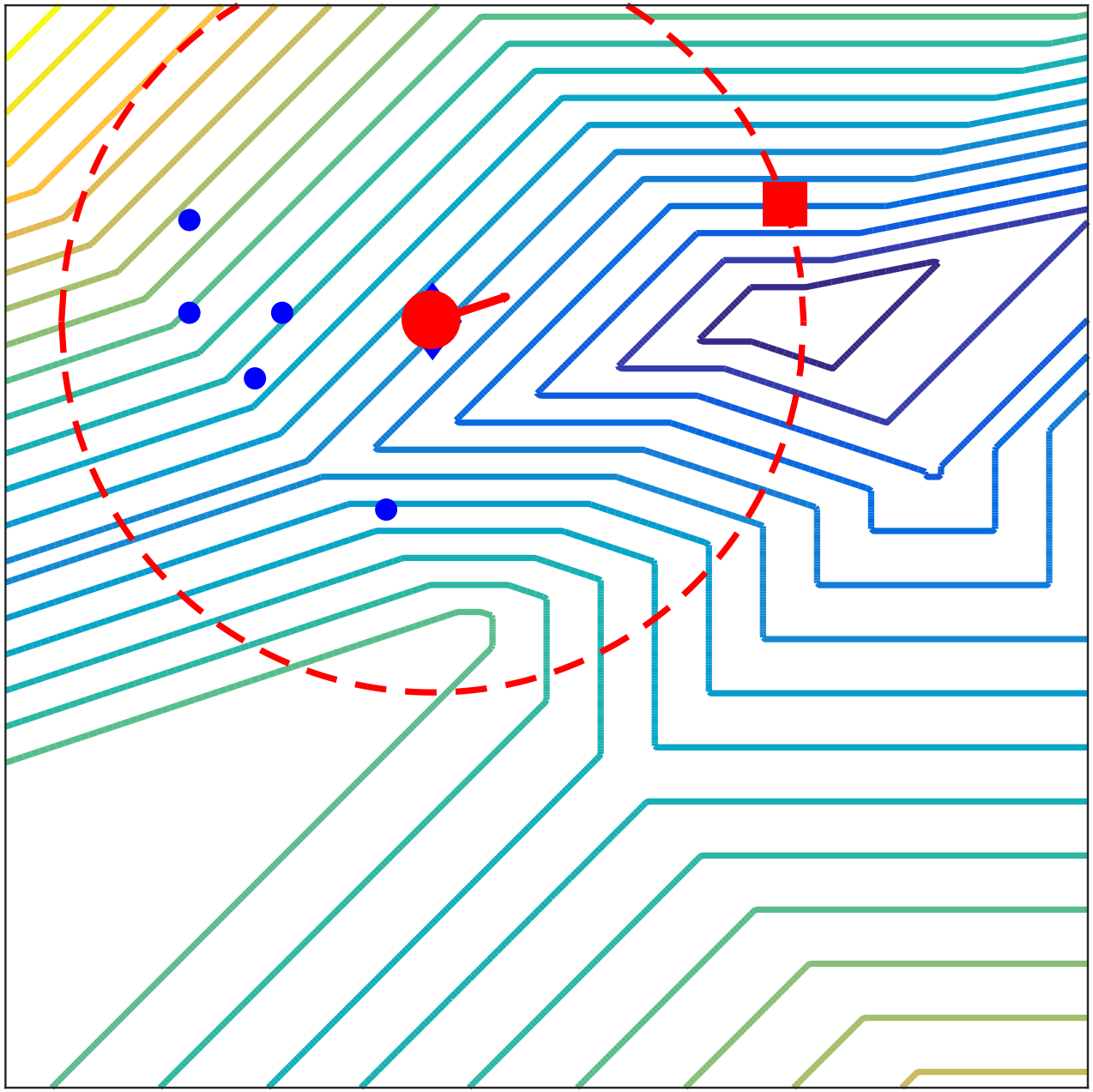

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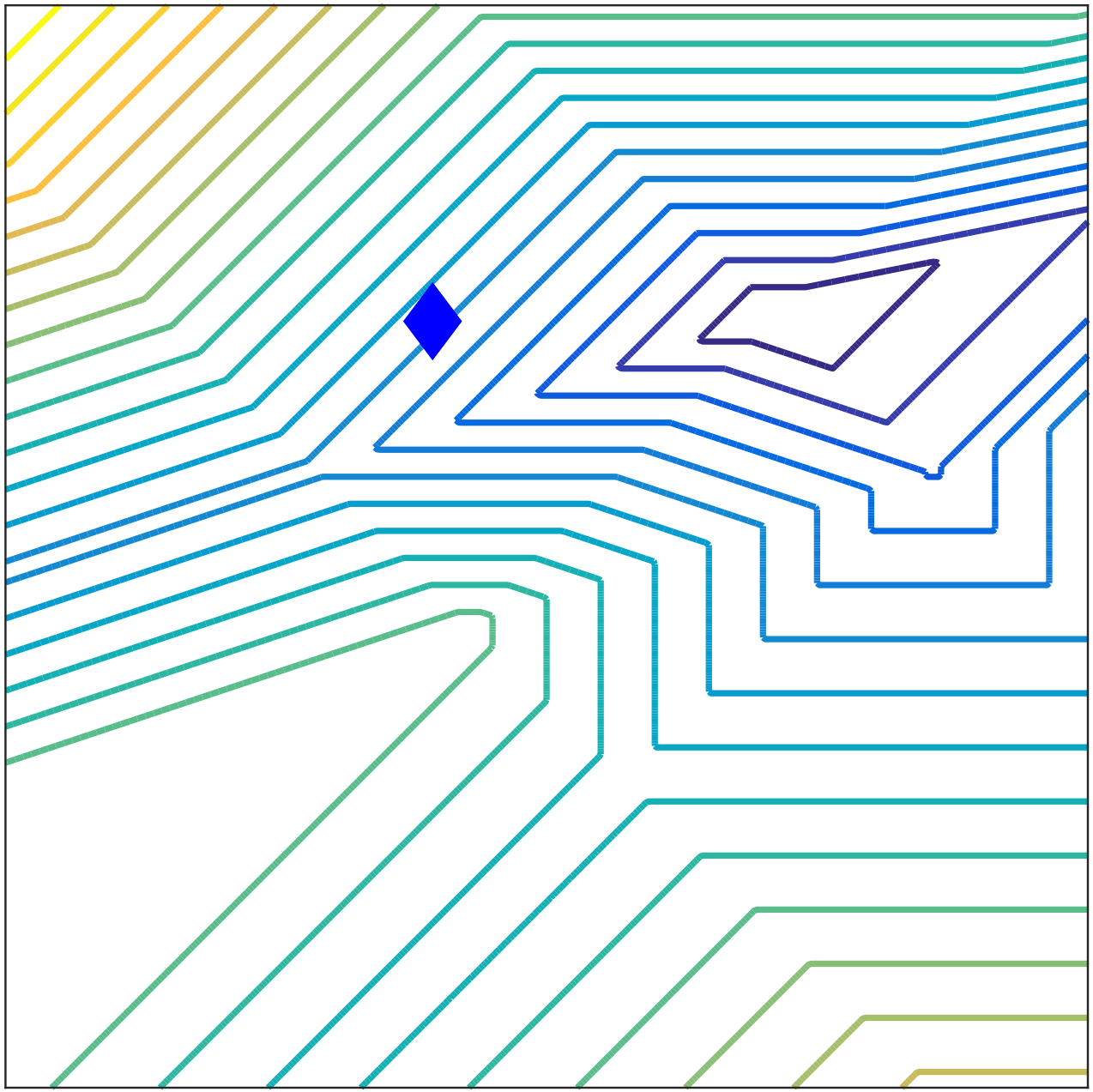

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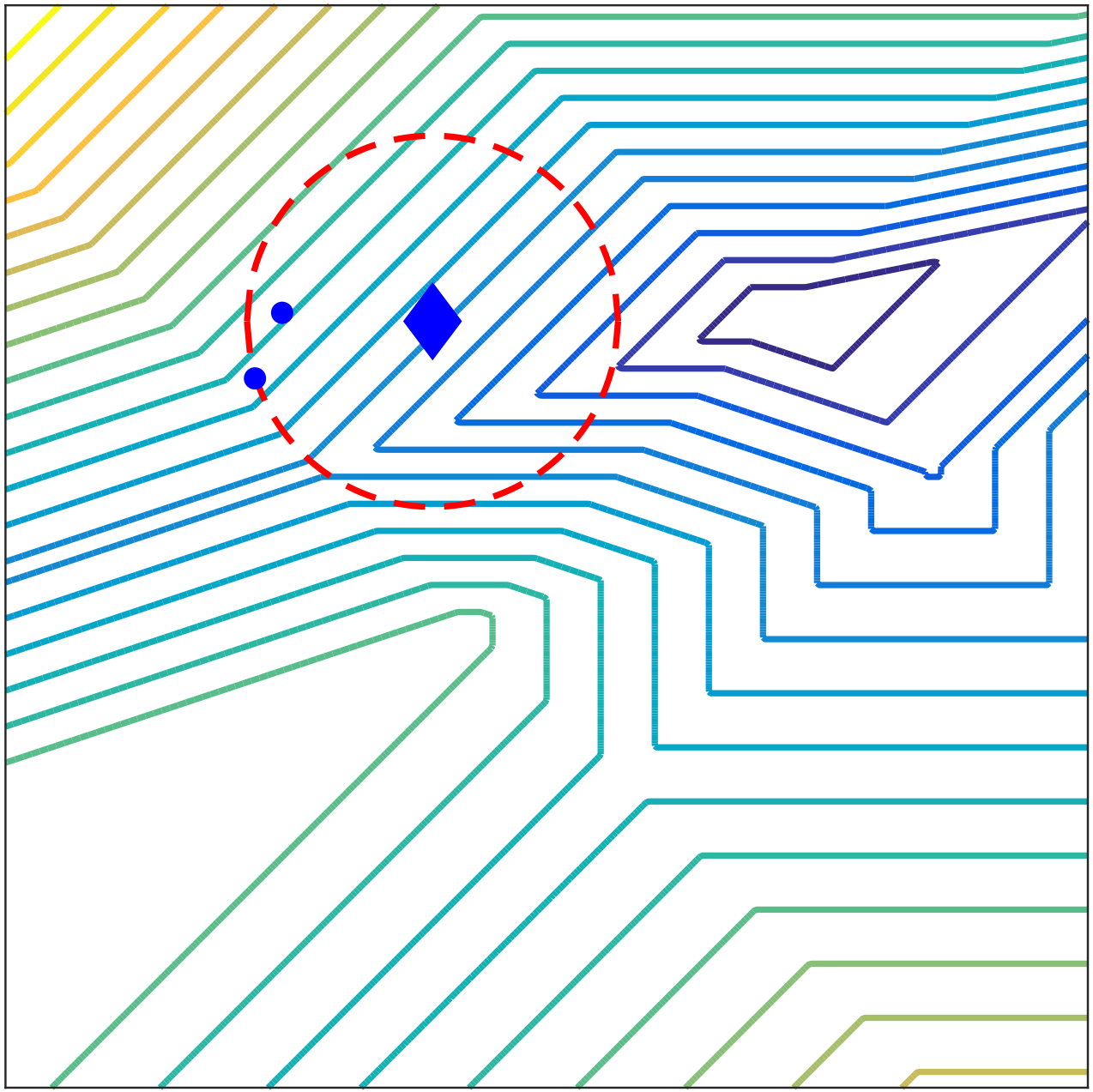

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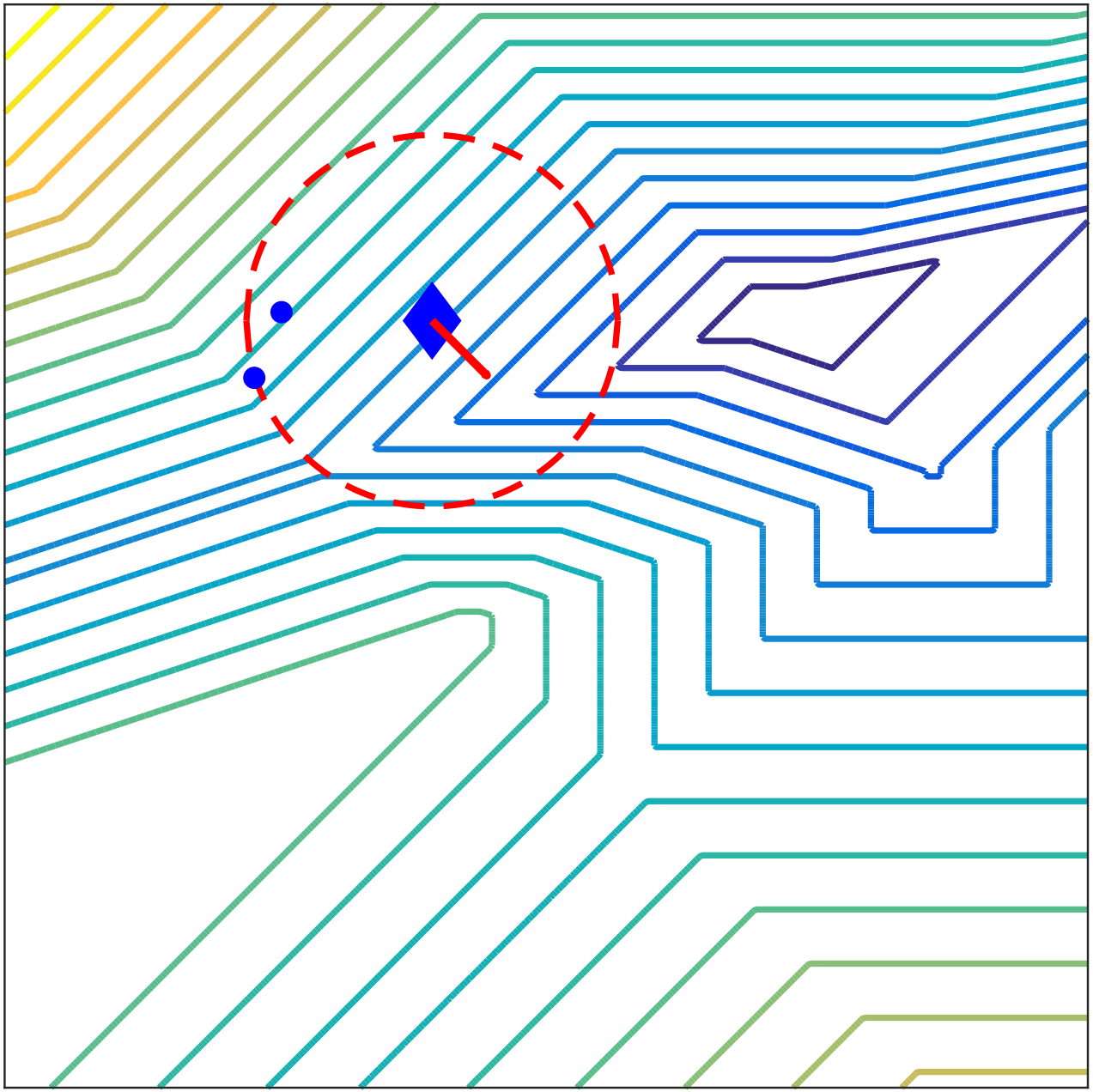

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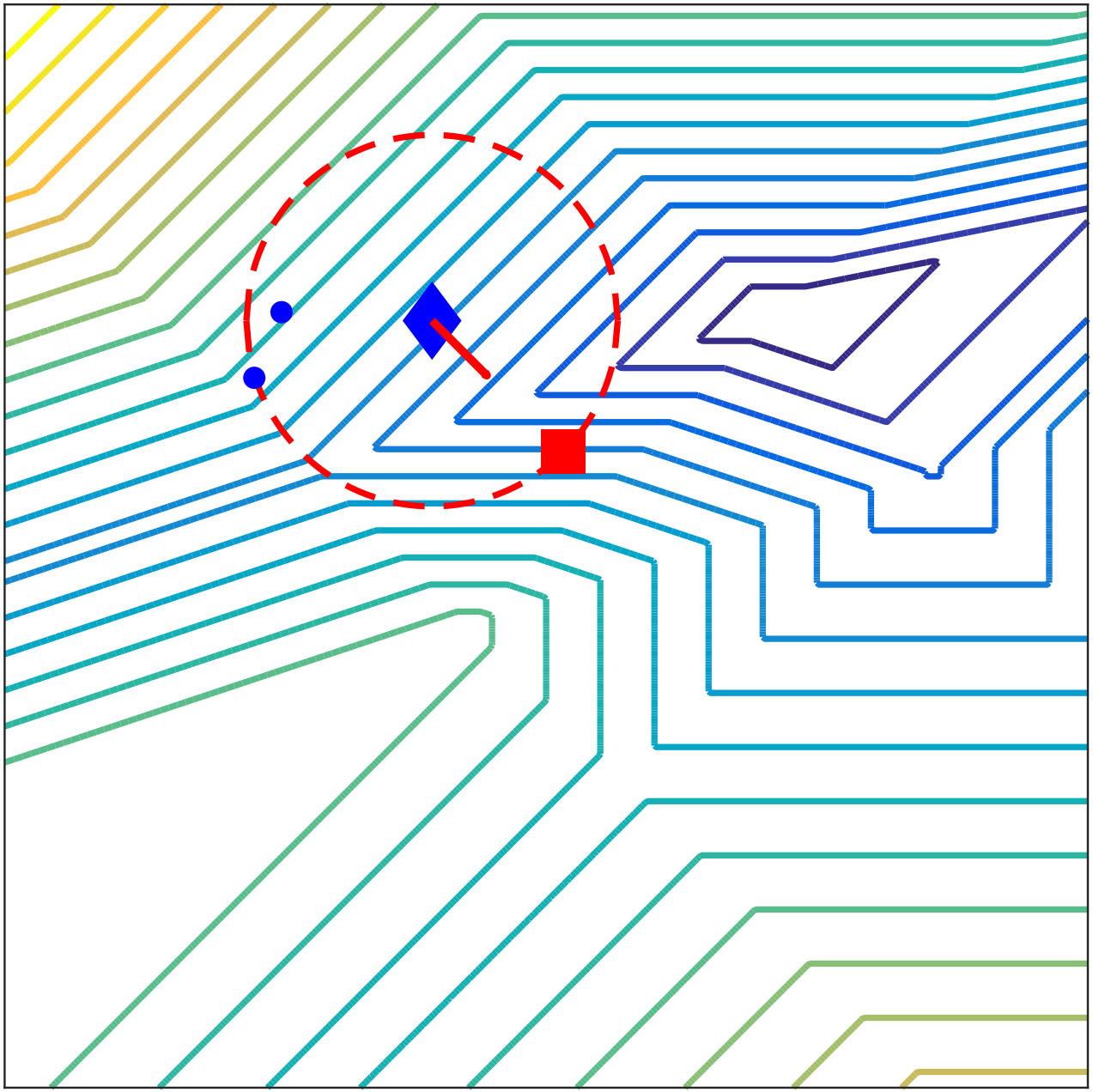

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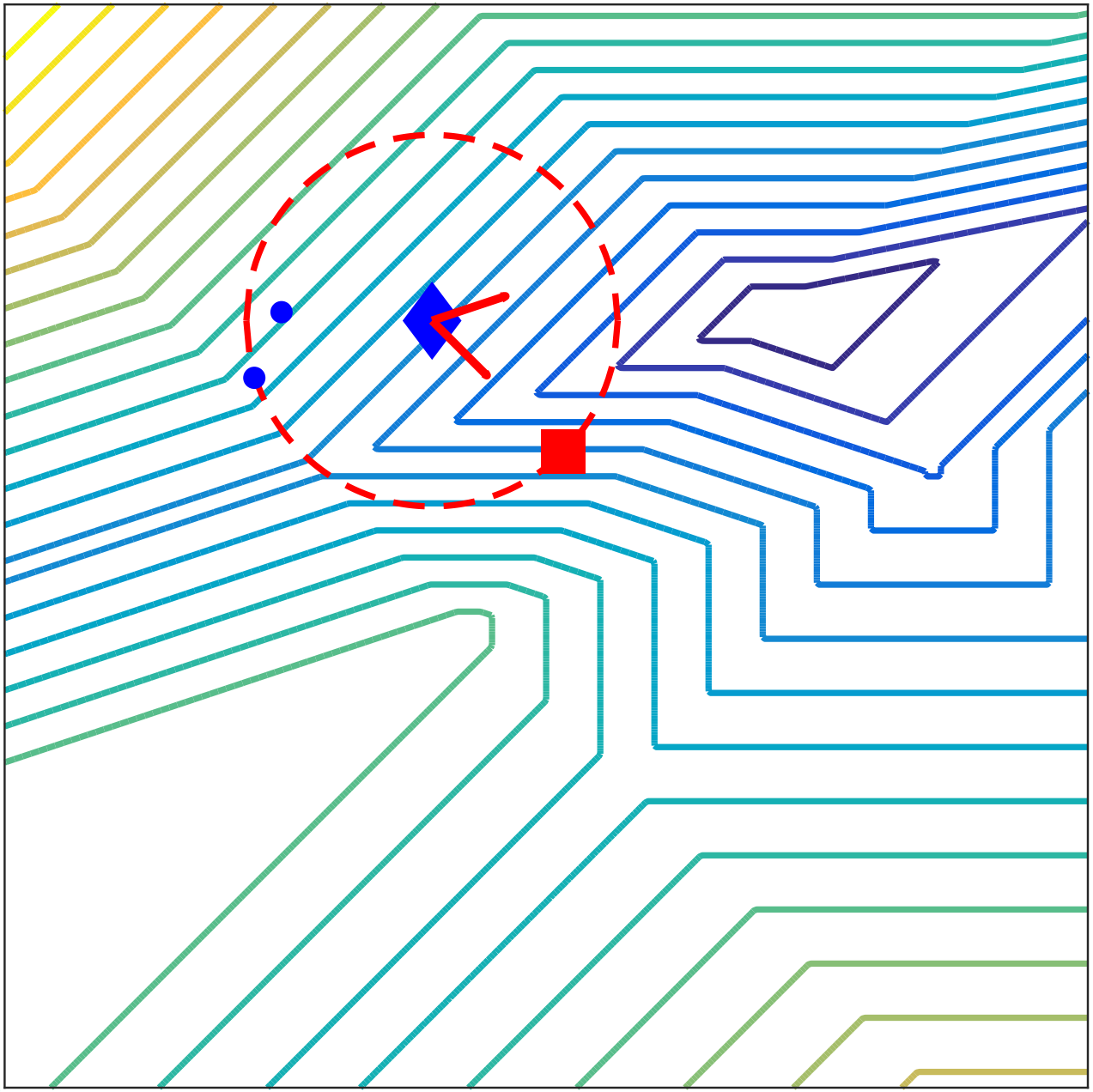

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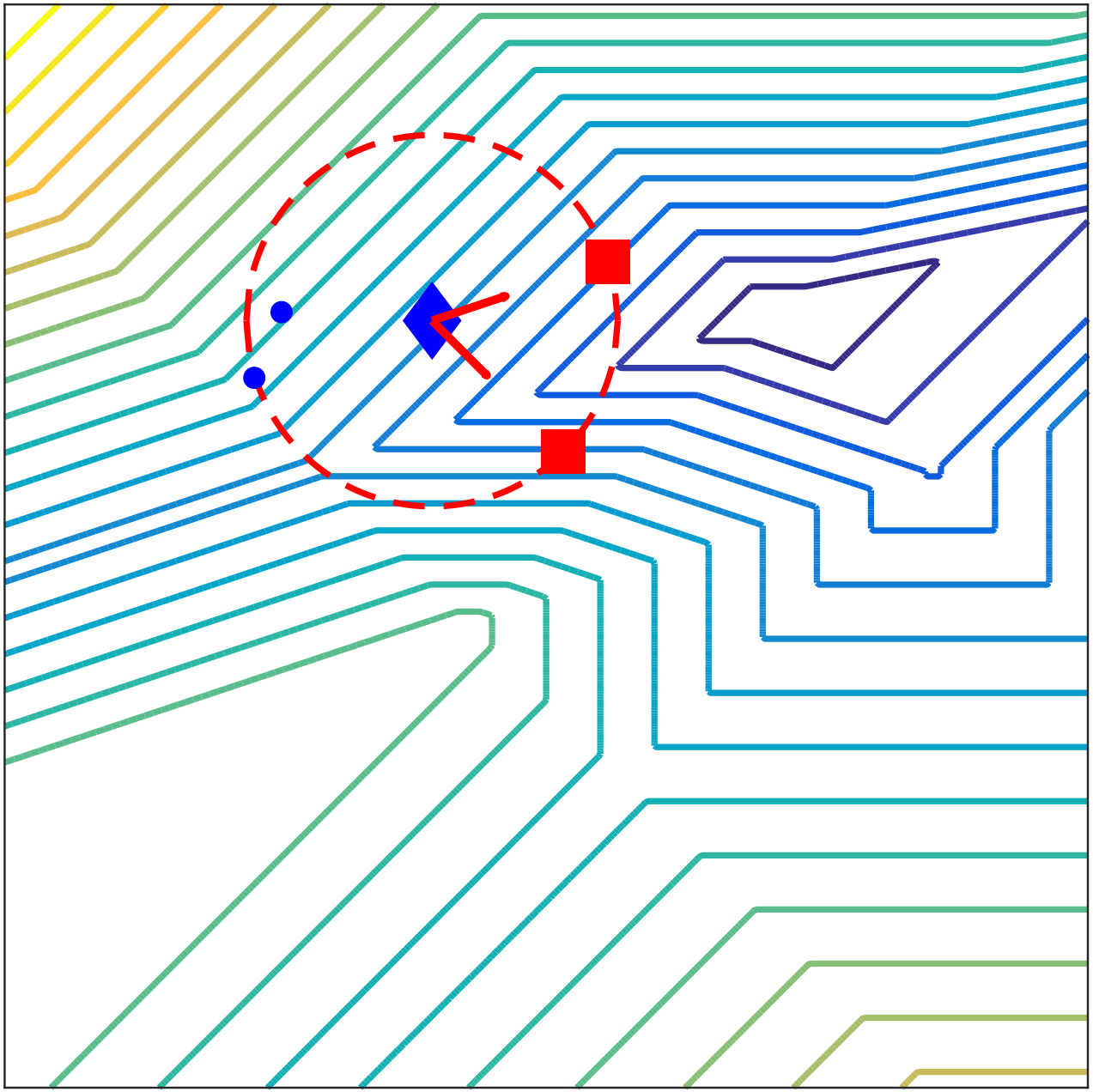

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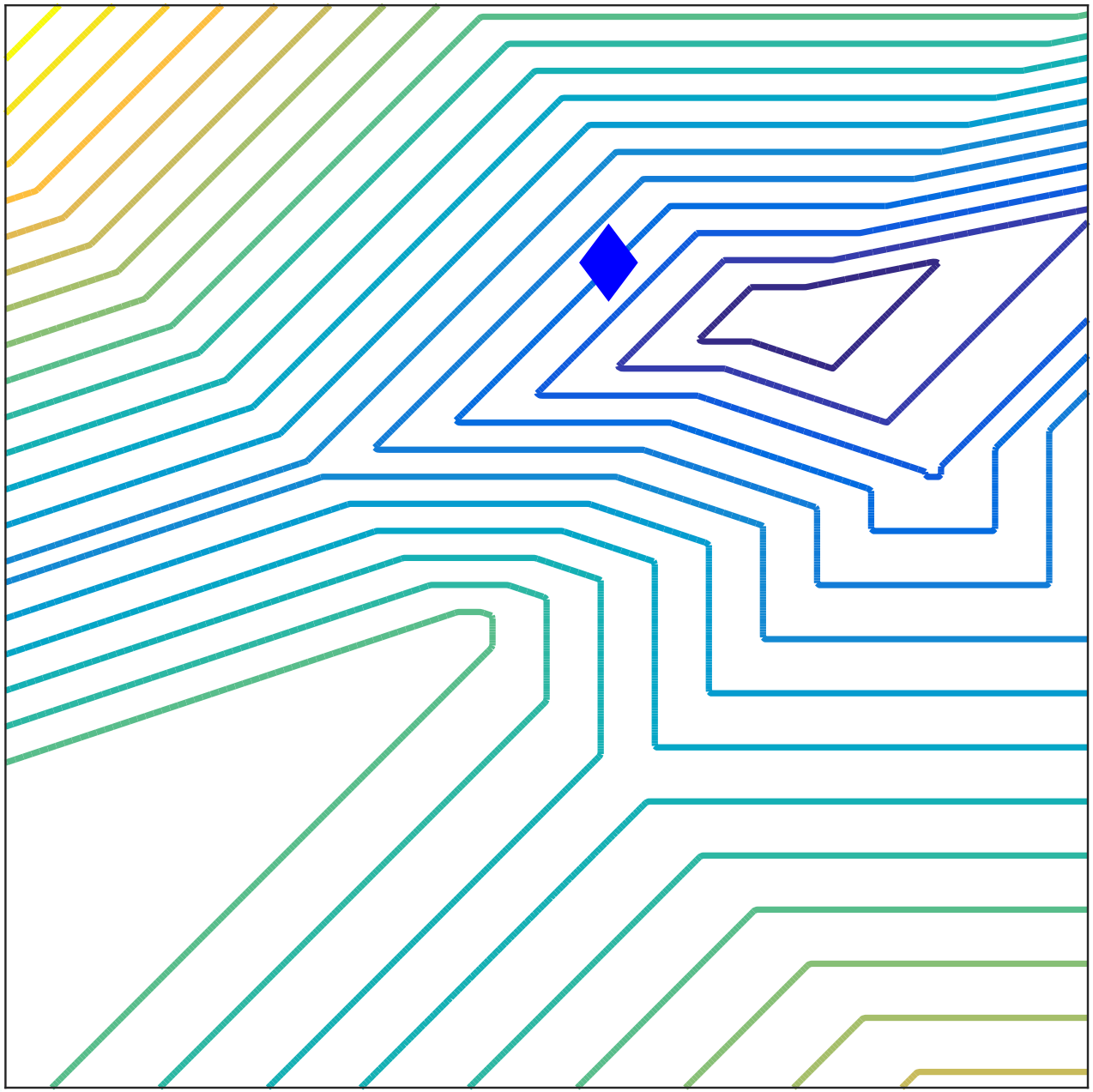

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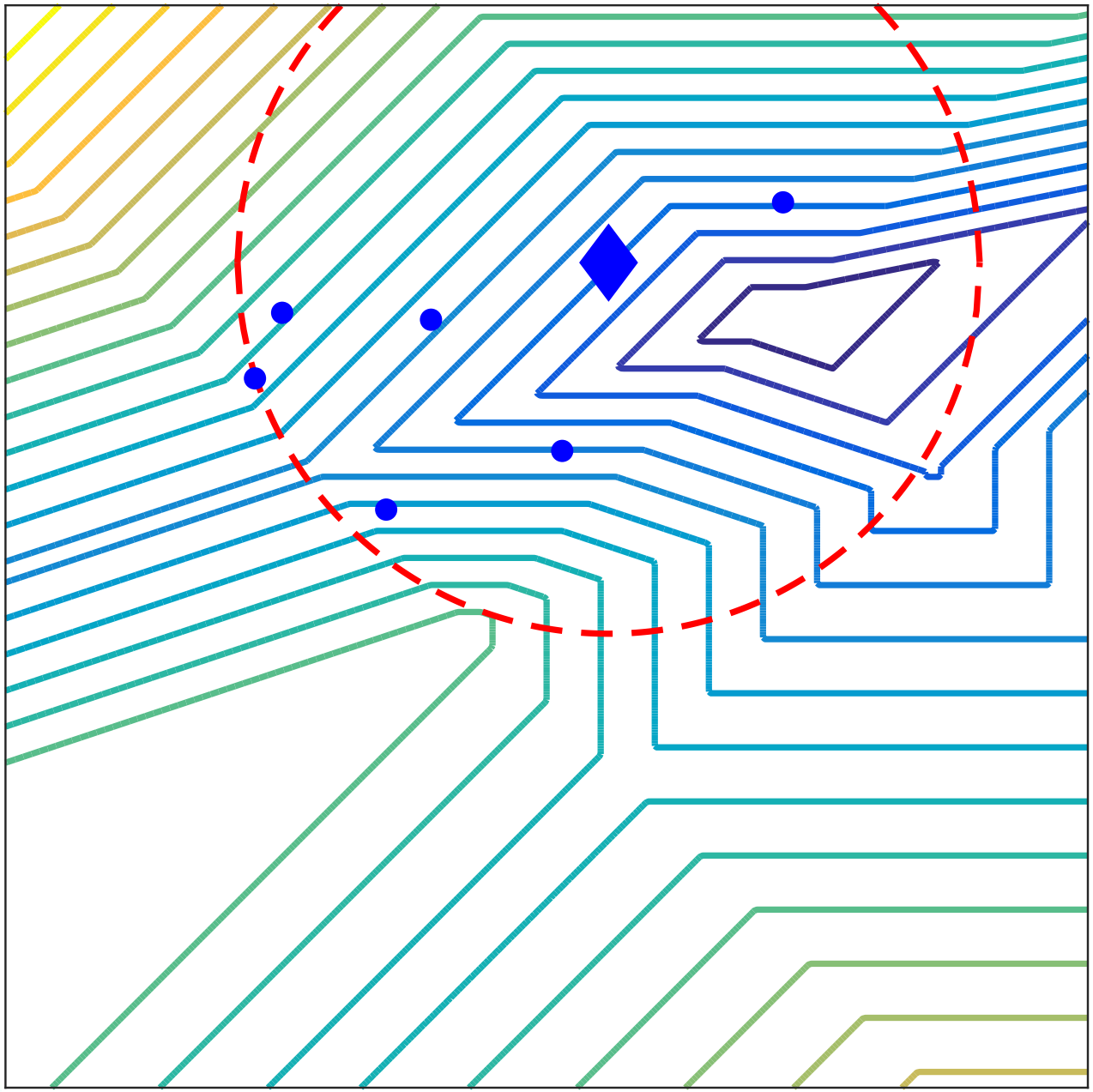

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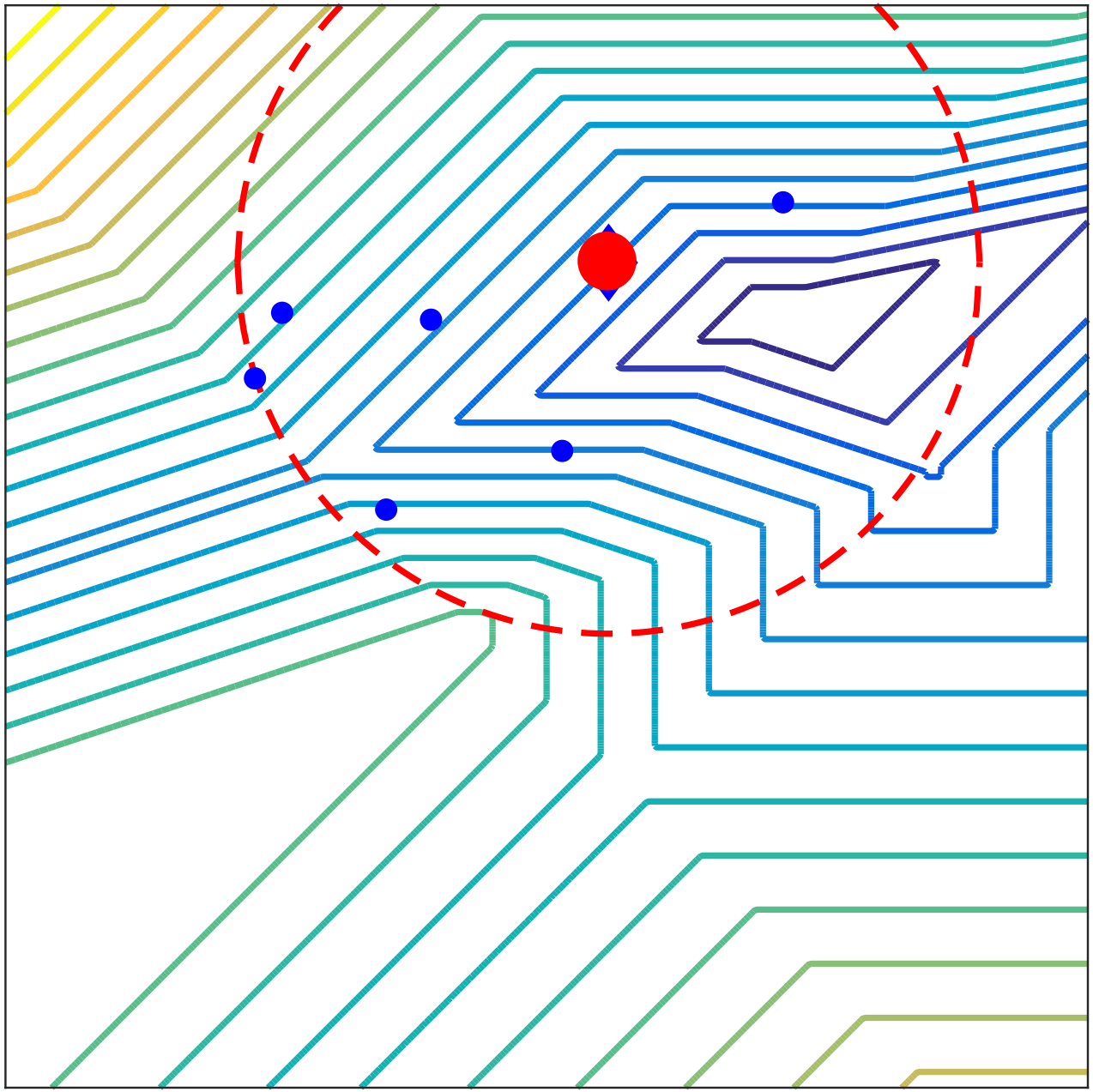

Better trust-region subproblem?

Instead of solving

$$\underset{s}{\text{minimize}}\ m_k^f(x^k + s)$$

$$\text{subject to: } s \in \mathcal{B}(0, \Delta_k)$$

How about

$$\underset{s}{\text{minimize}}\ h(M(x^k + s))$$

$$\text{subject to: } s \in \mathcal{B}(0, \Delta_k)$$

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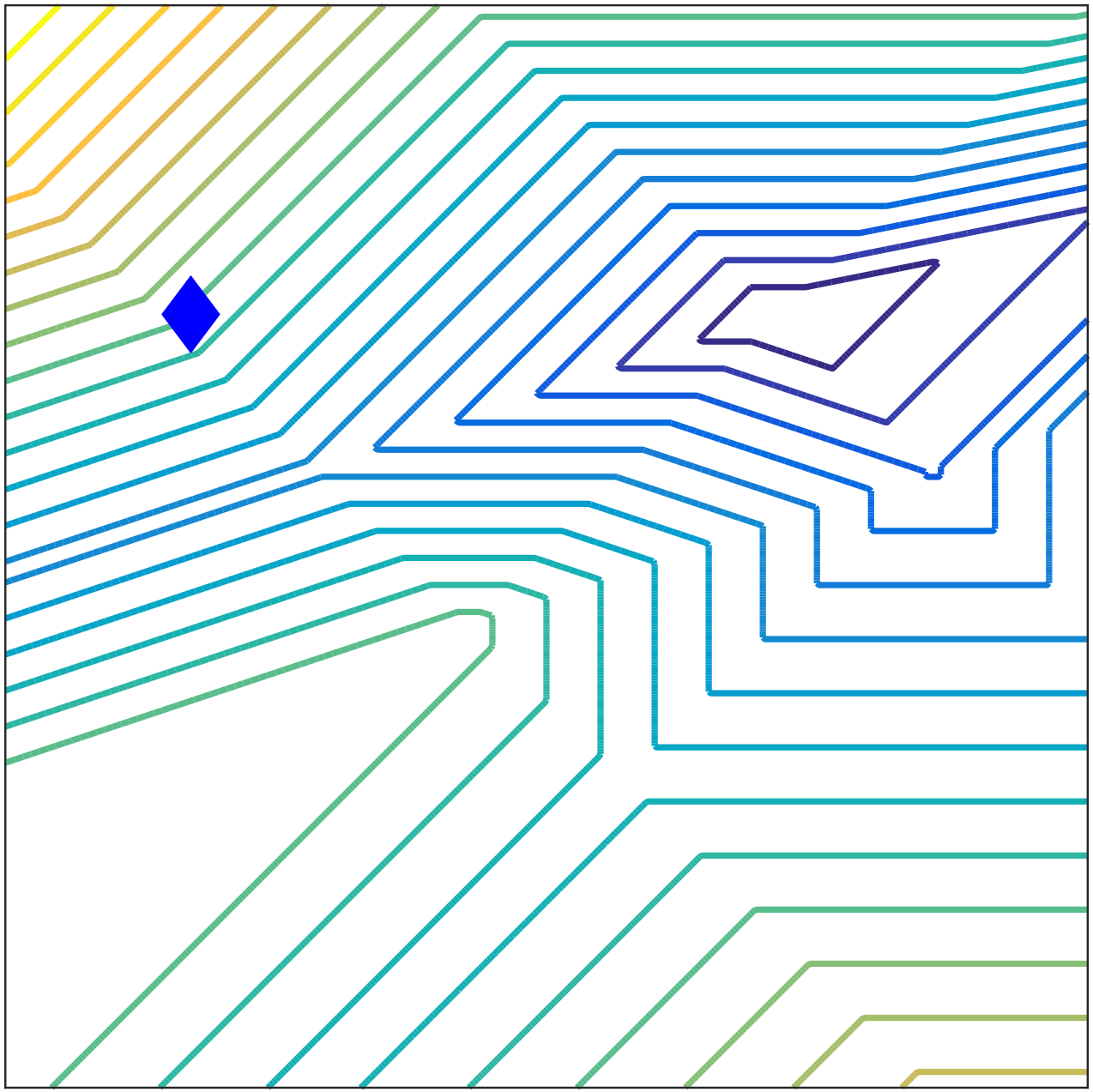

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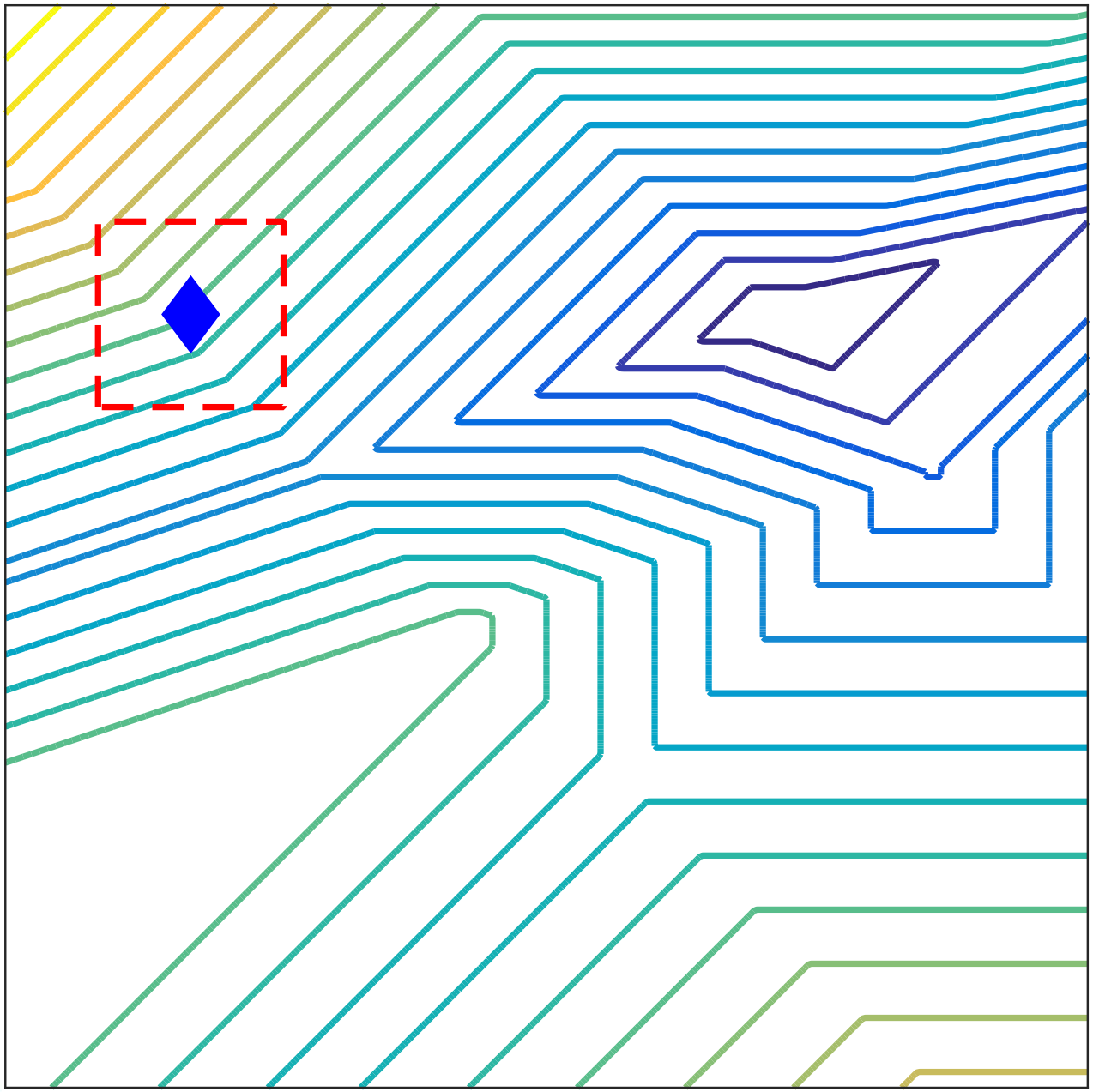

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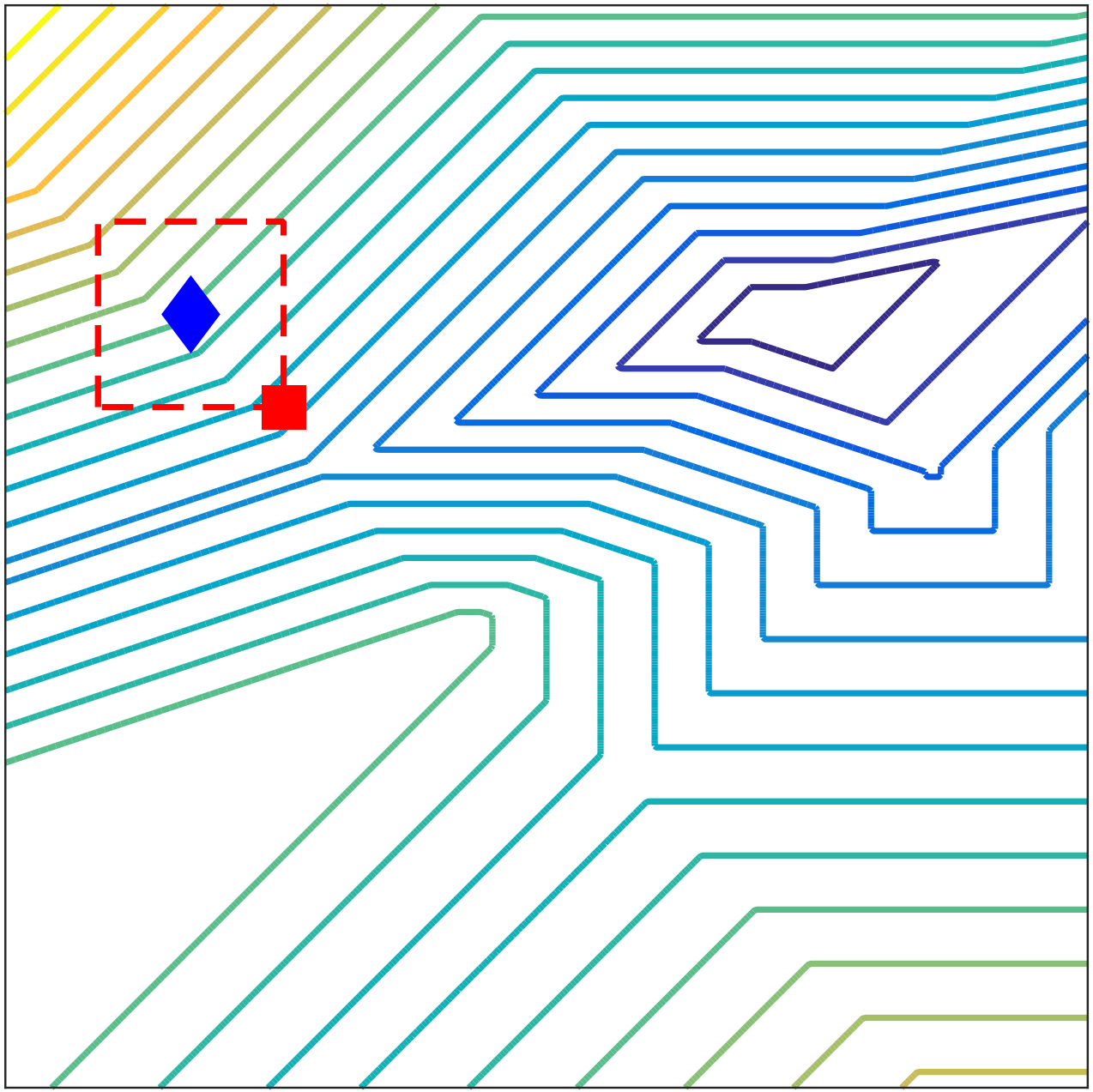

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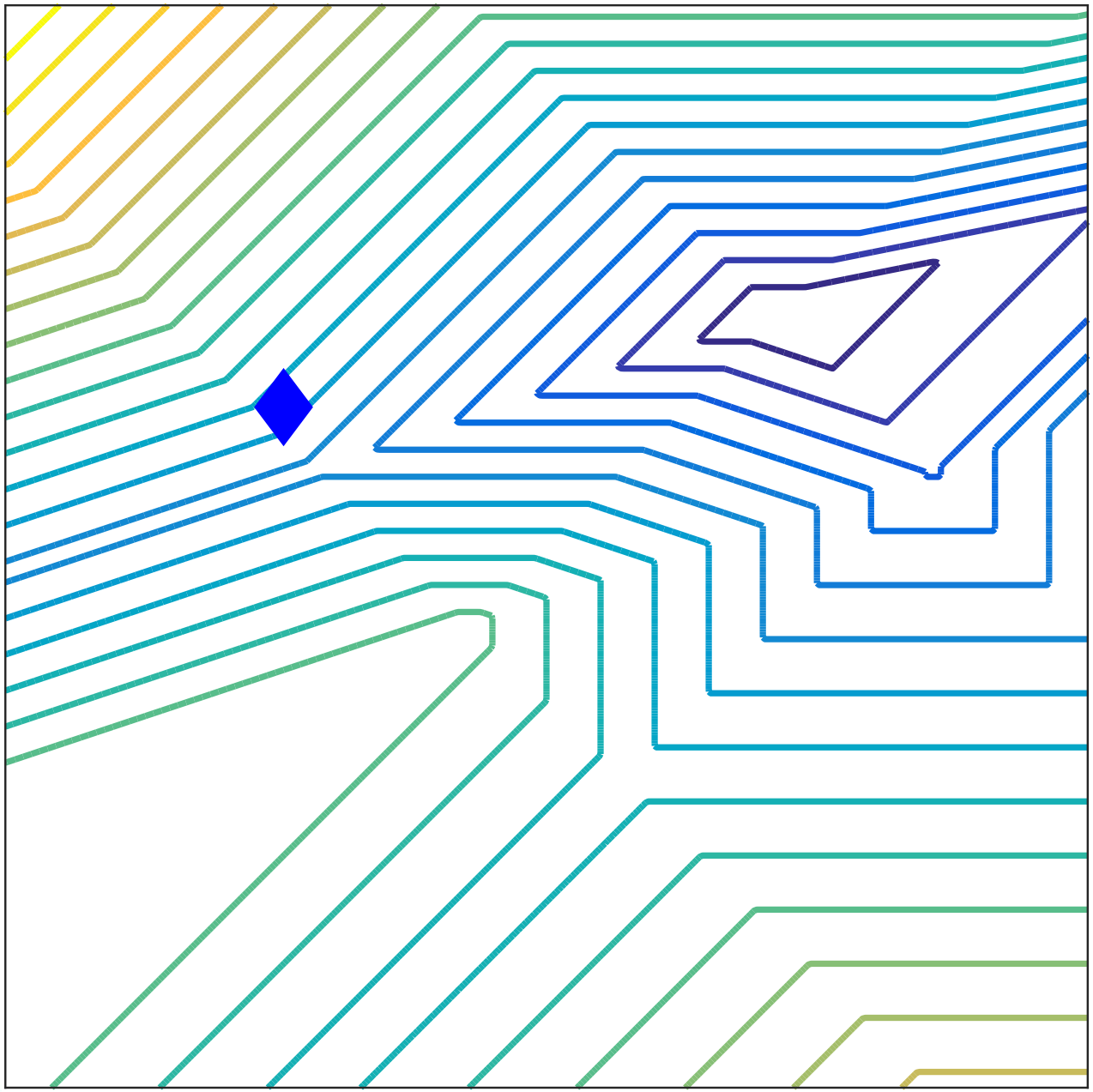

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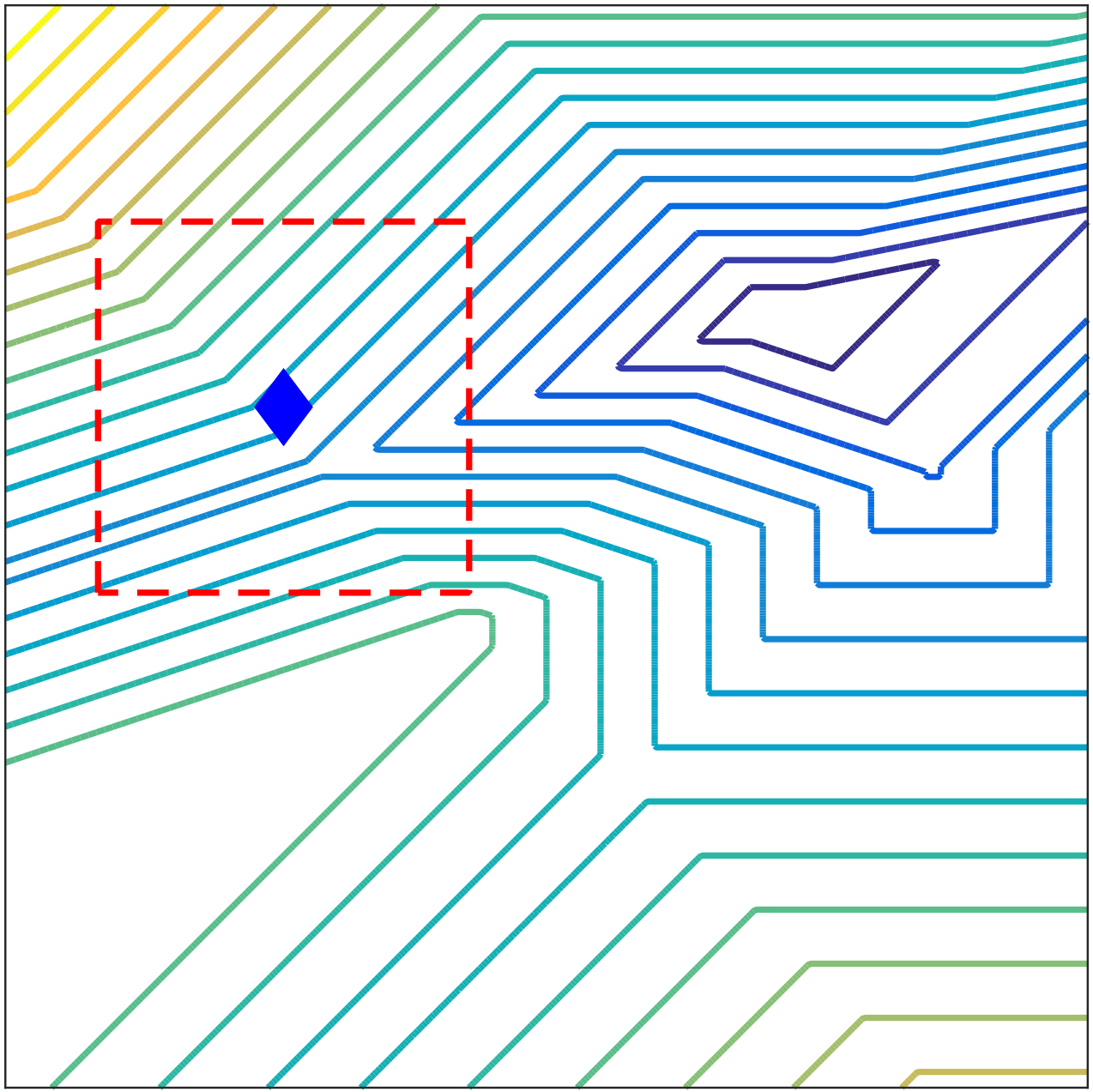

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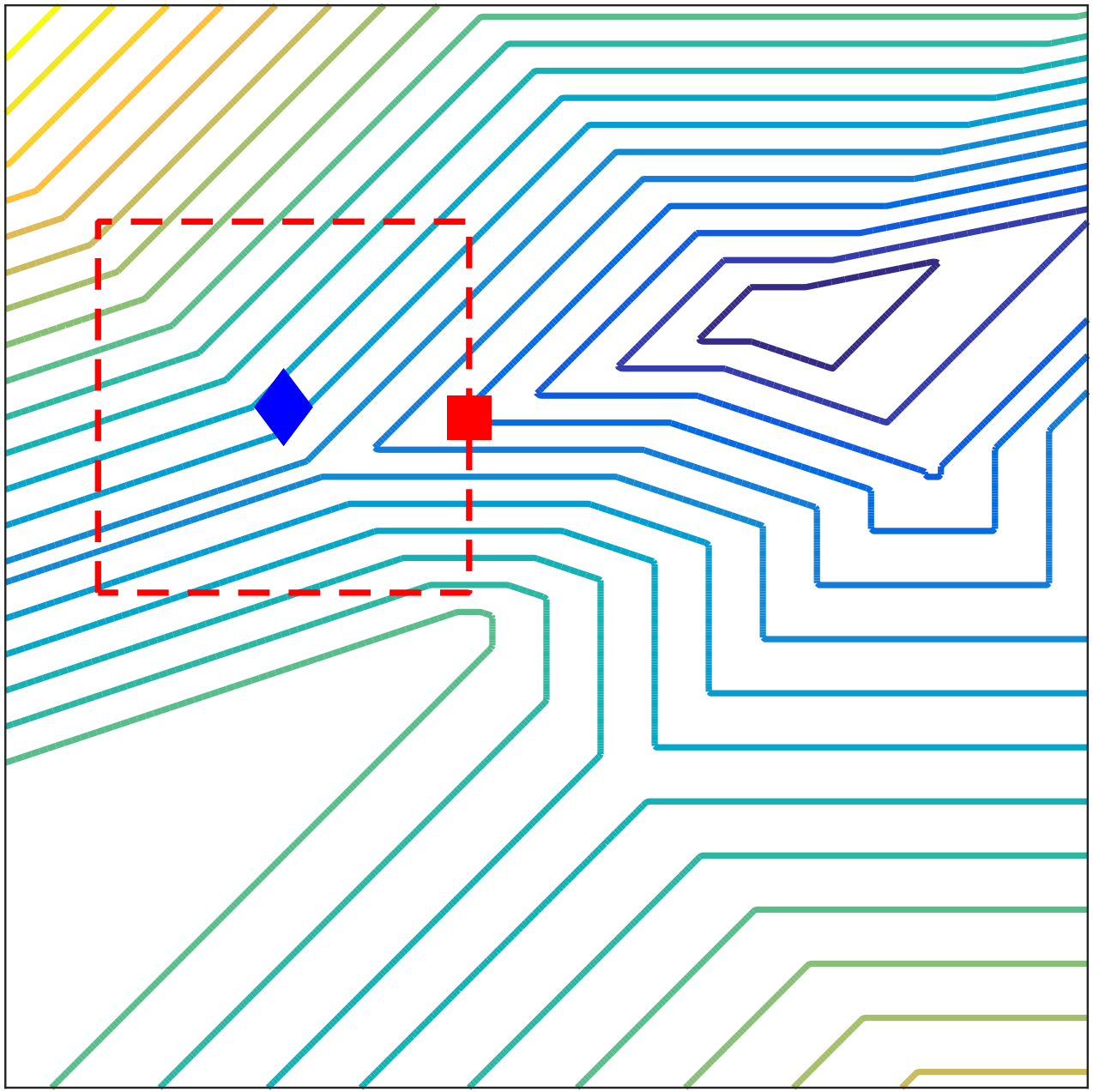

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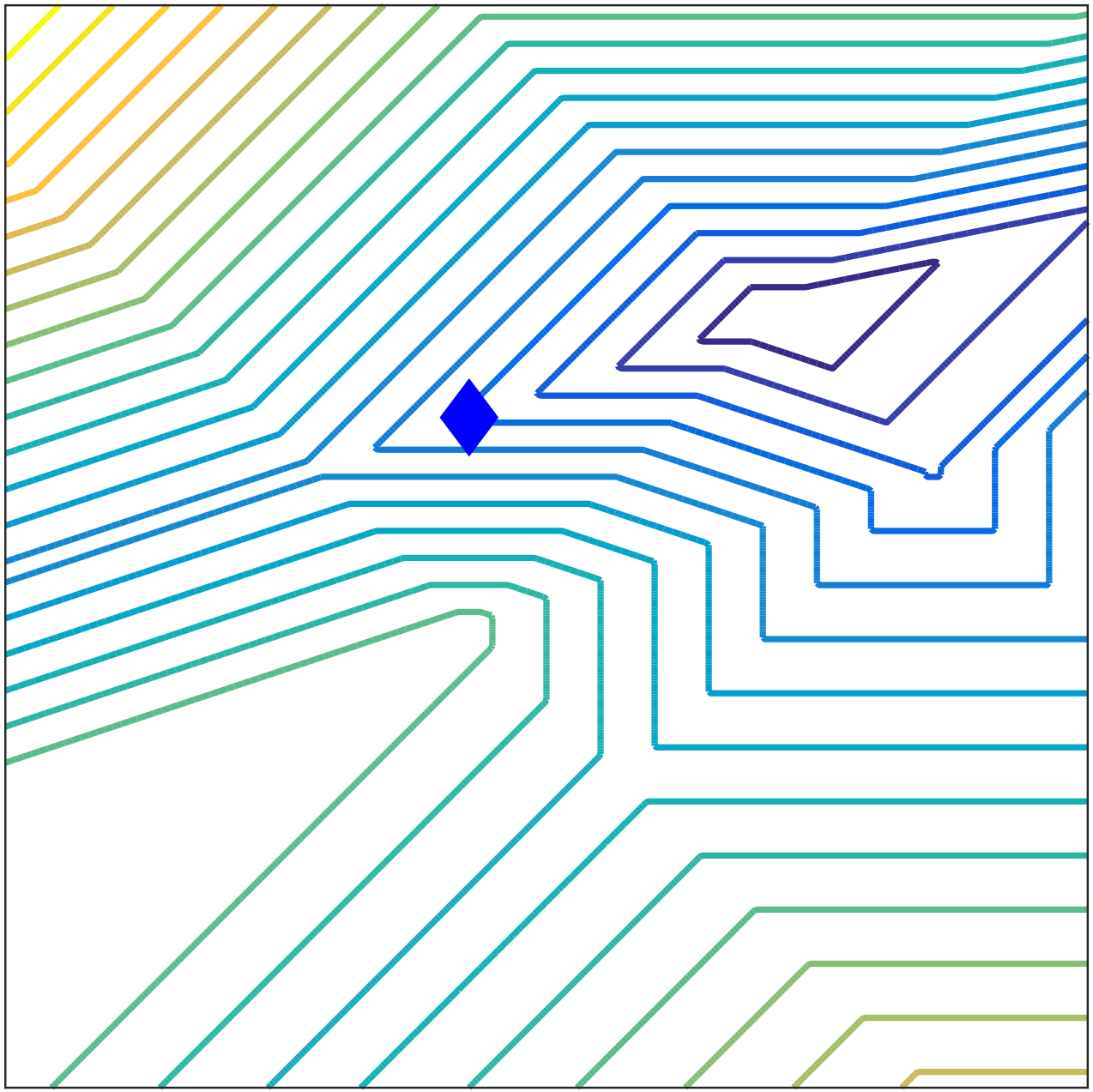

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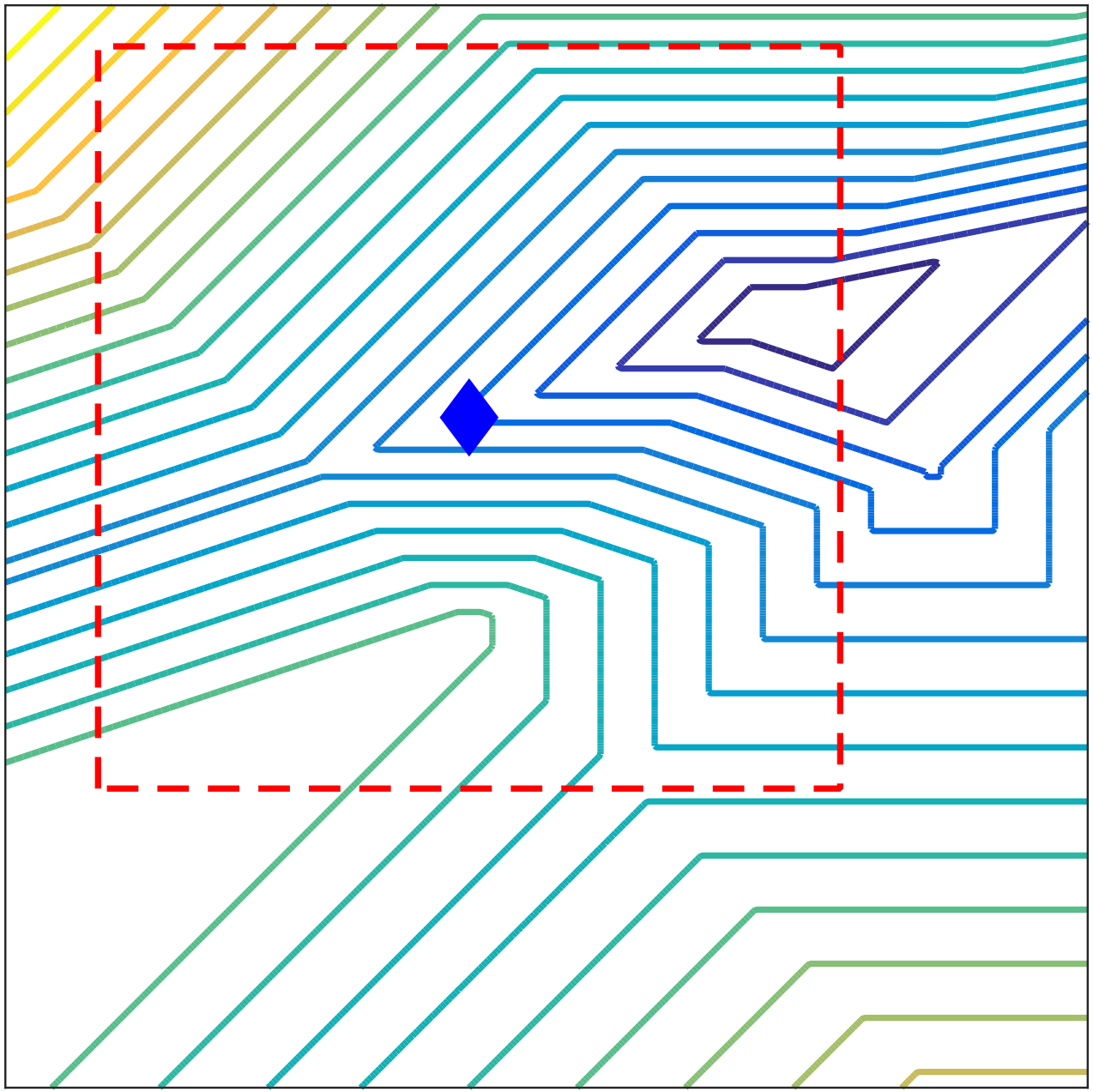

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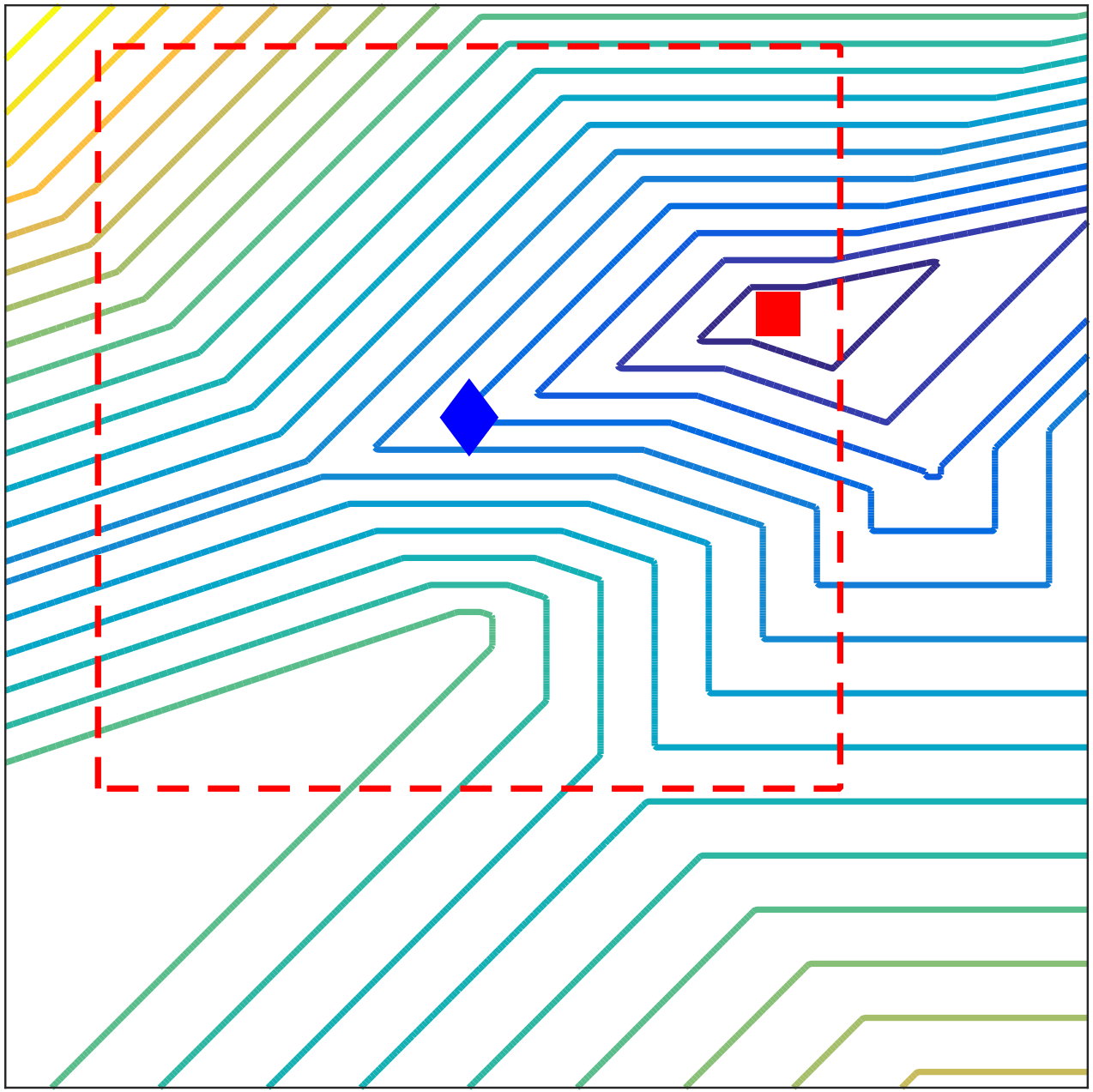

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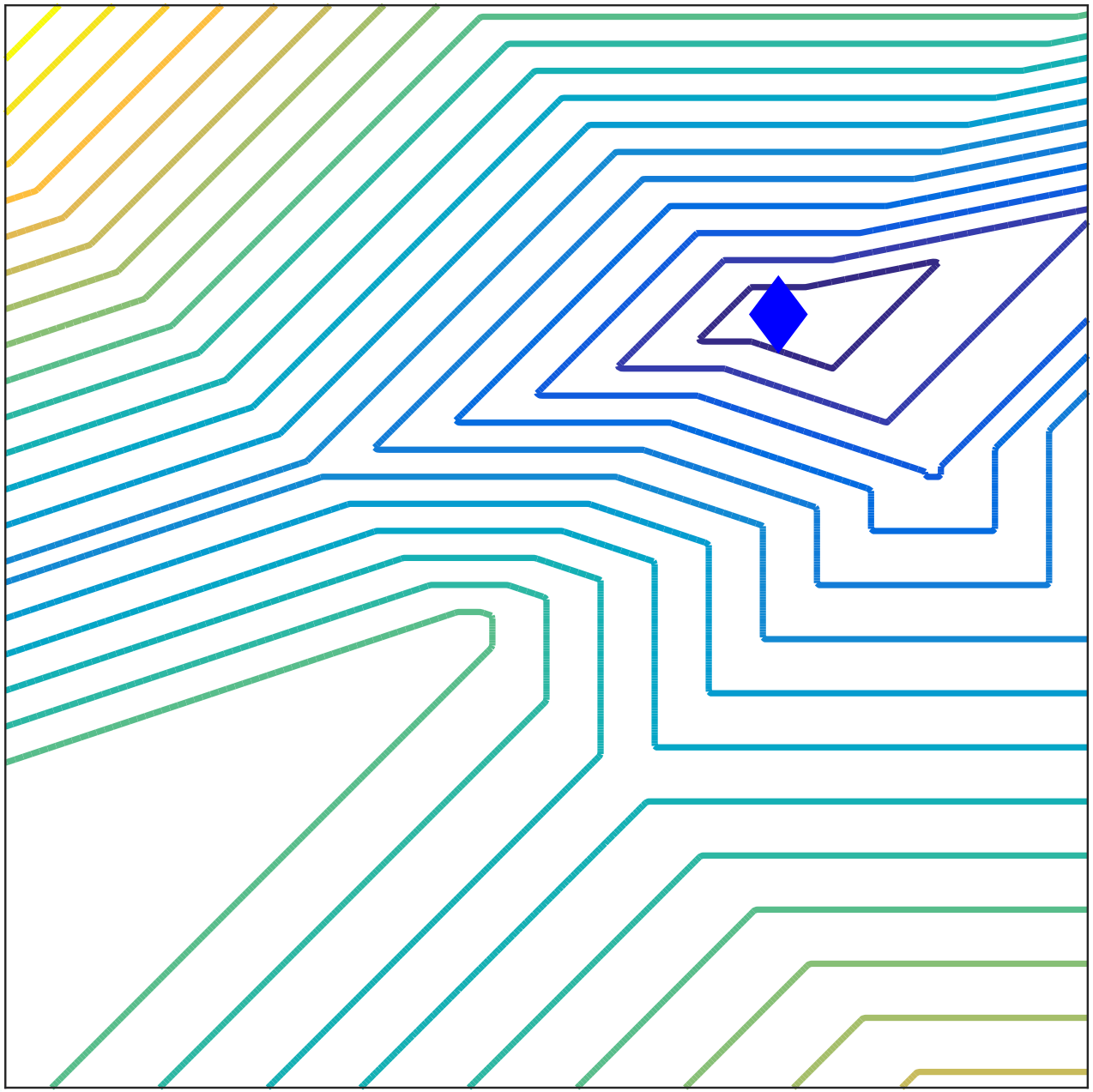

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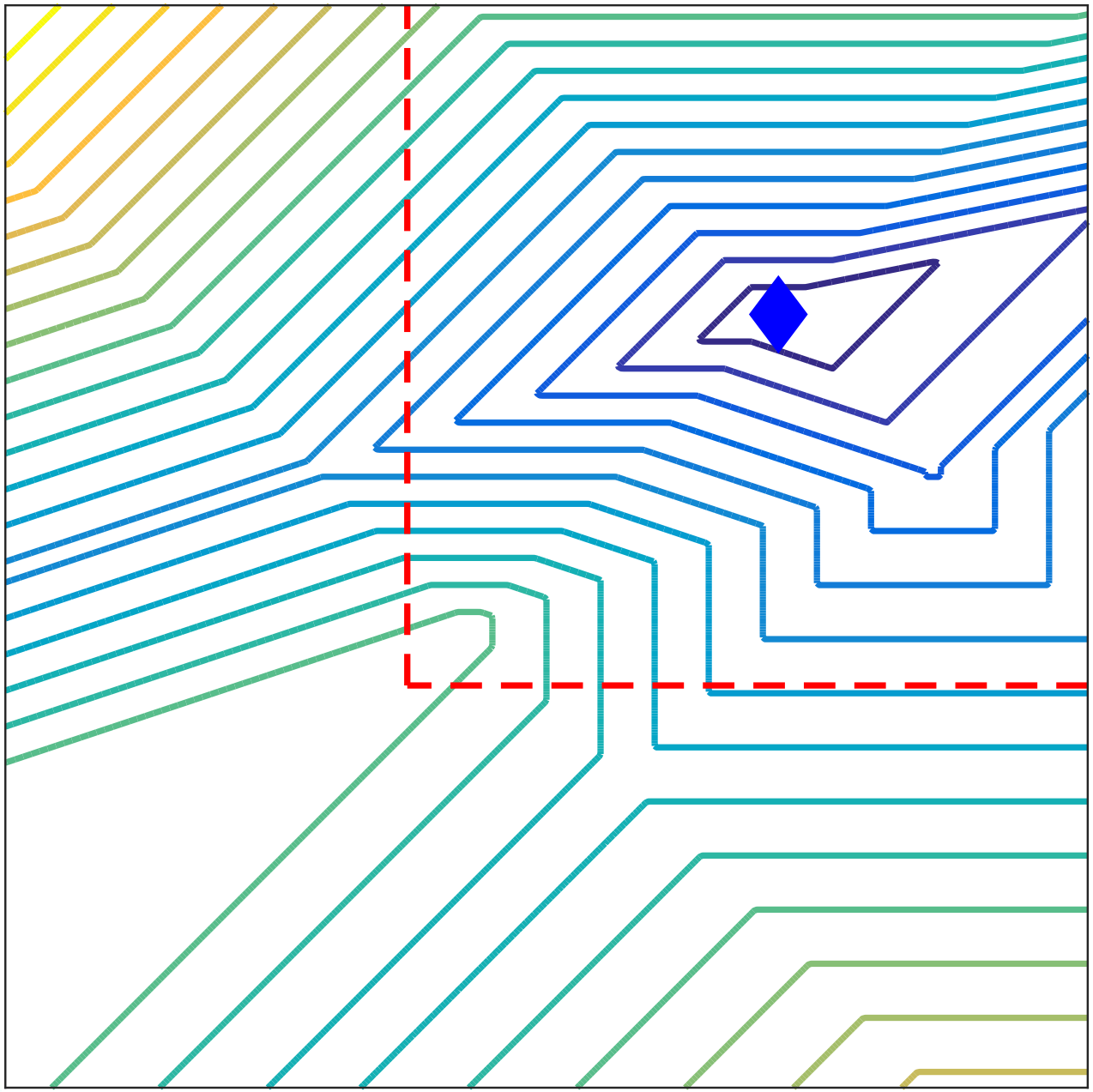

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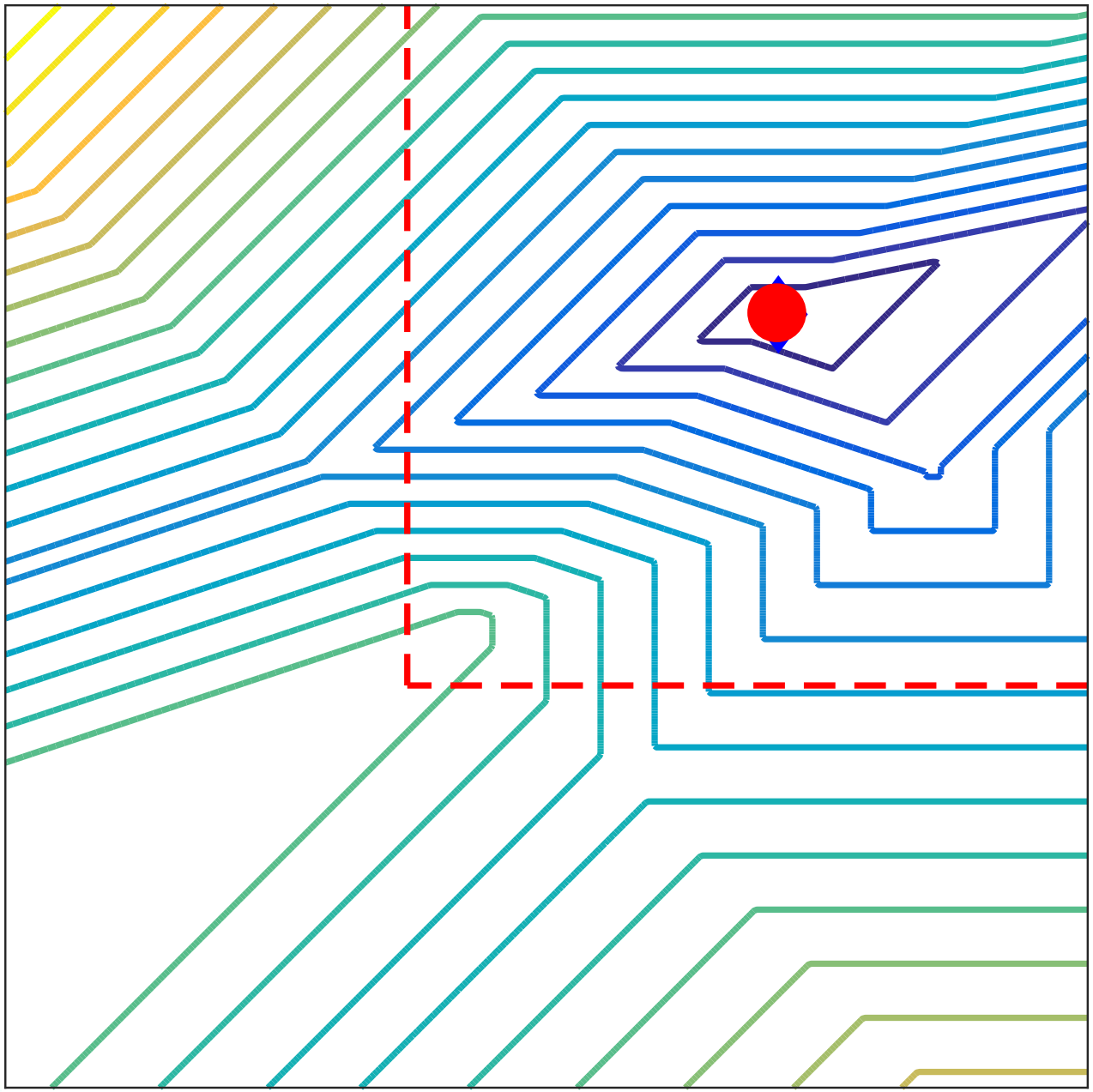

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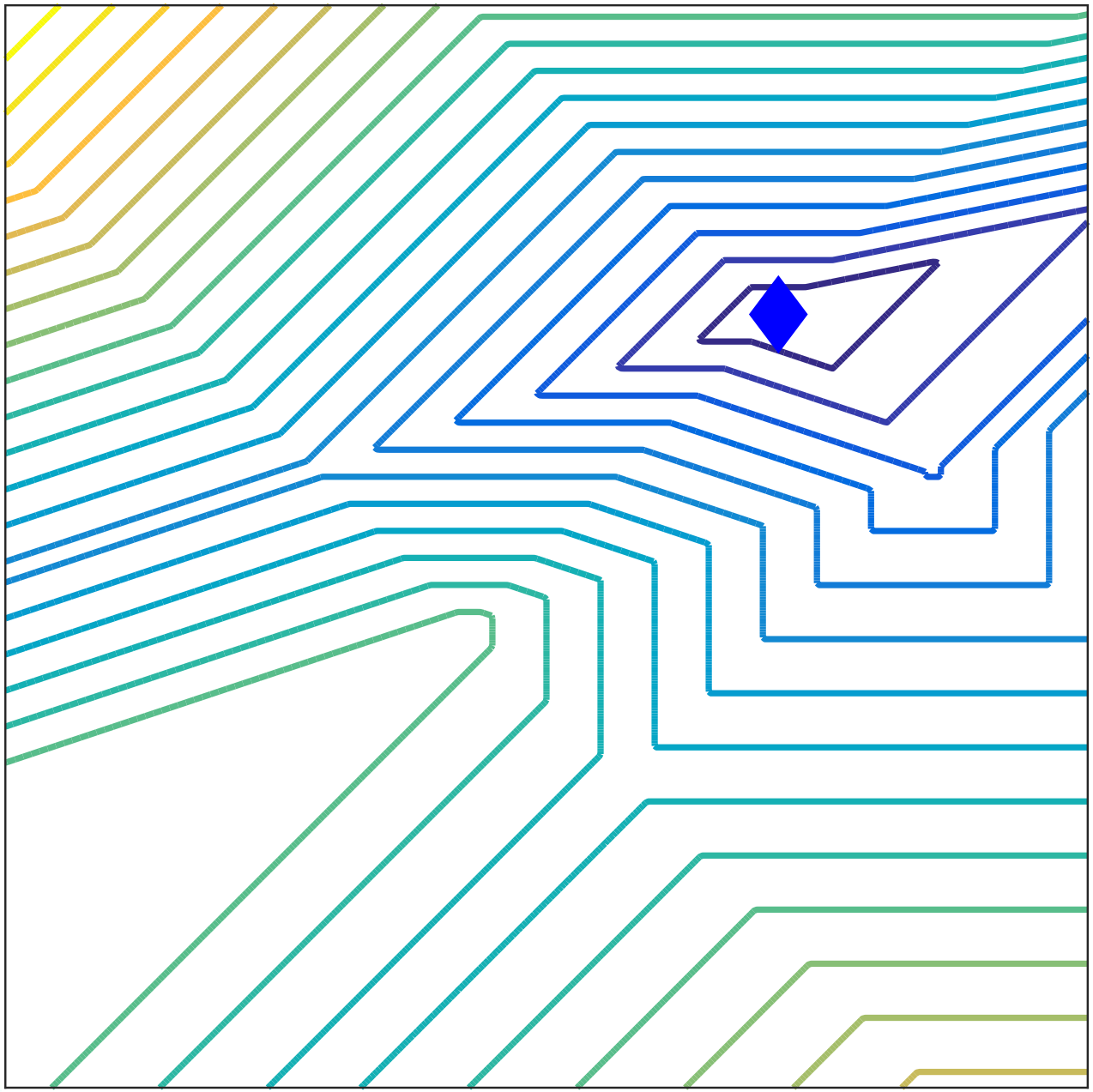

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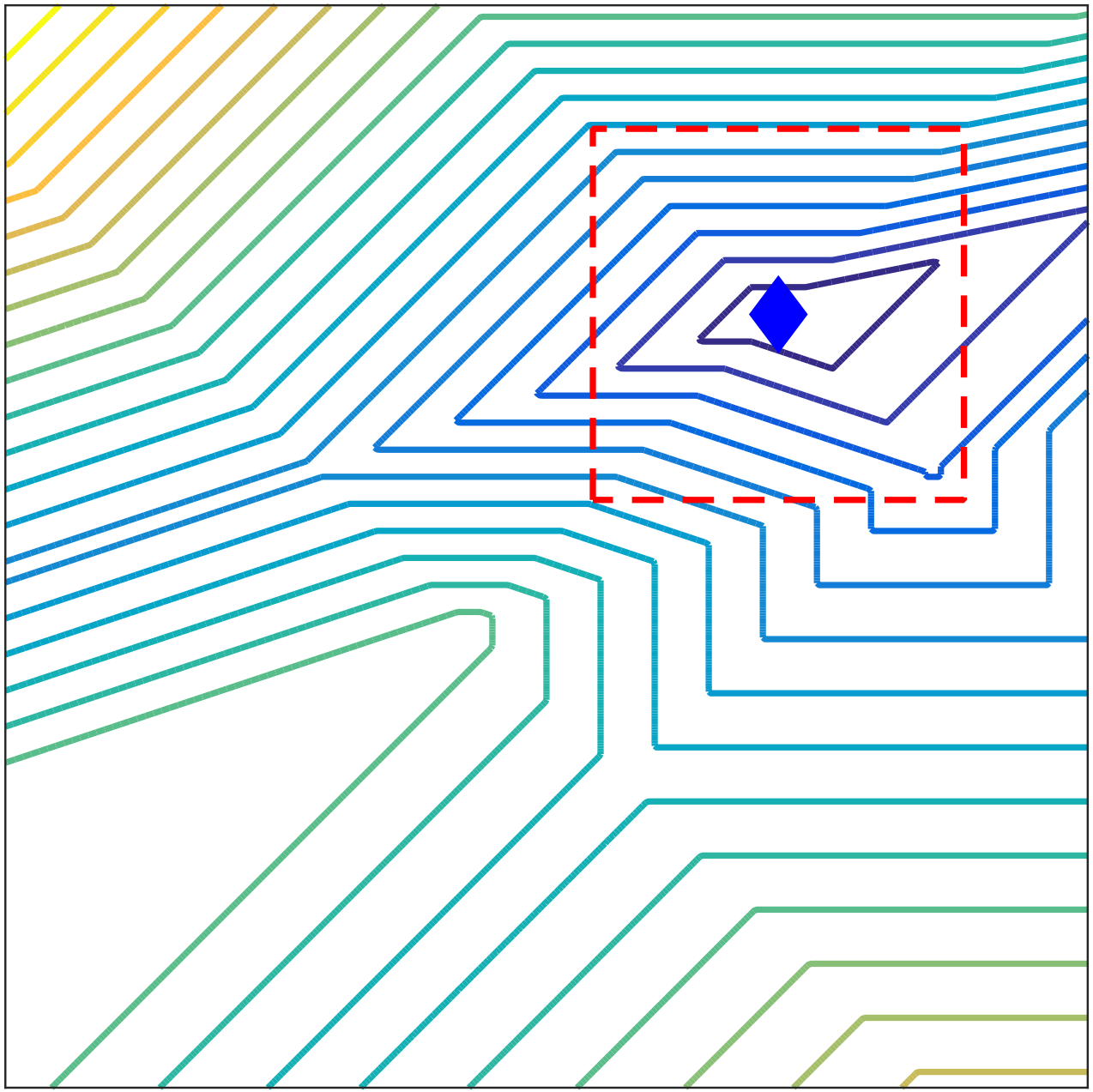

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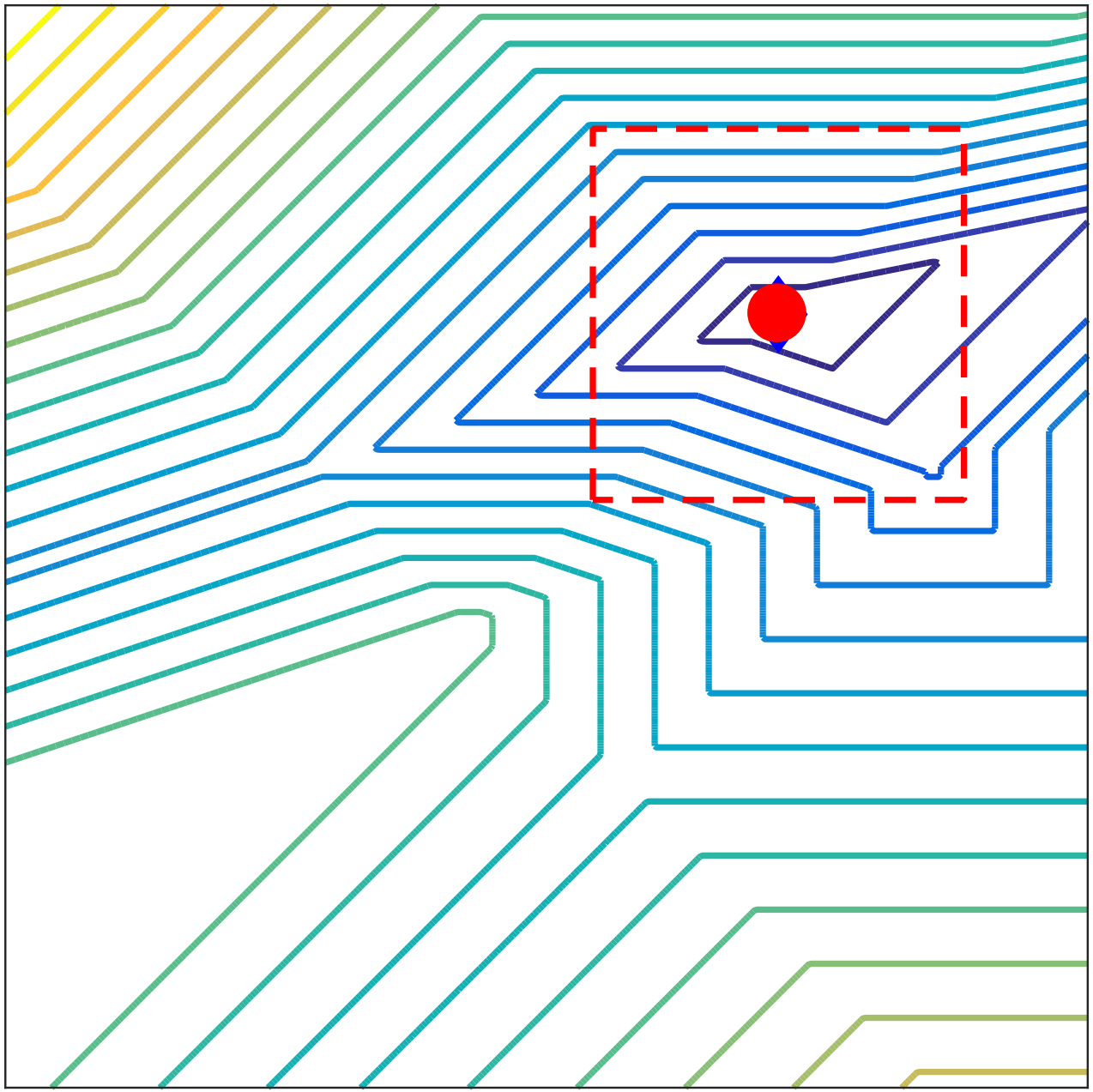

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Question

Best method for solving composite nonsmooth quadratic problems?

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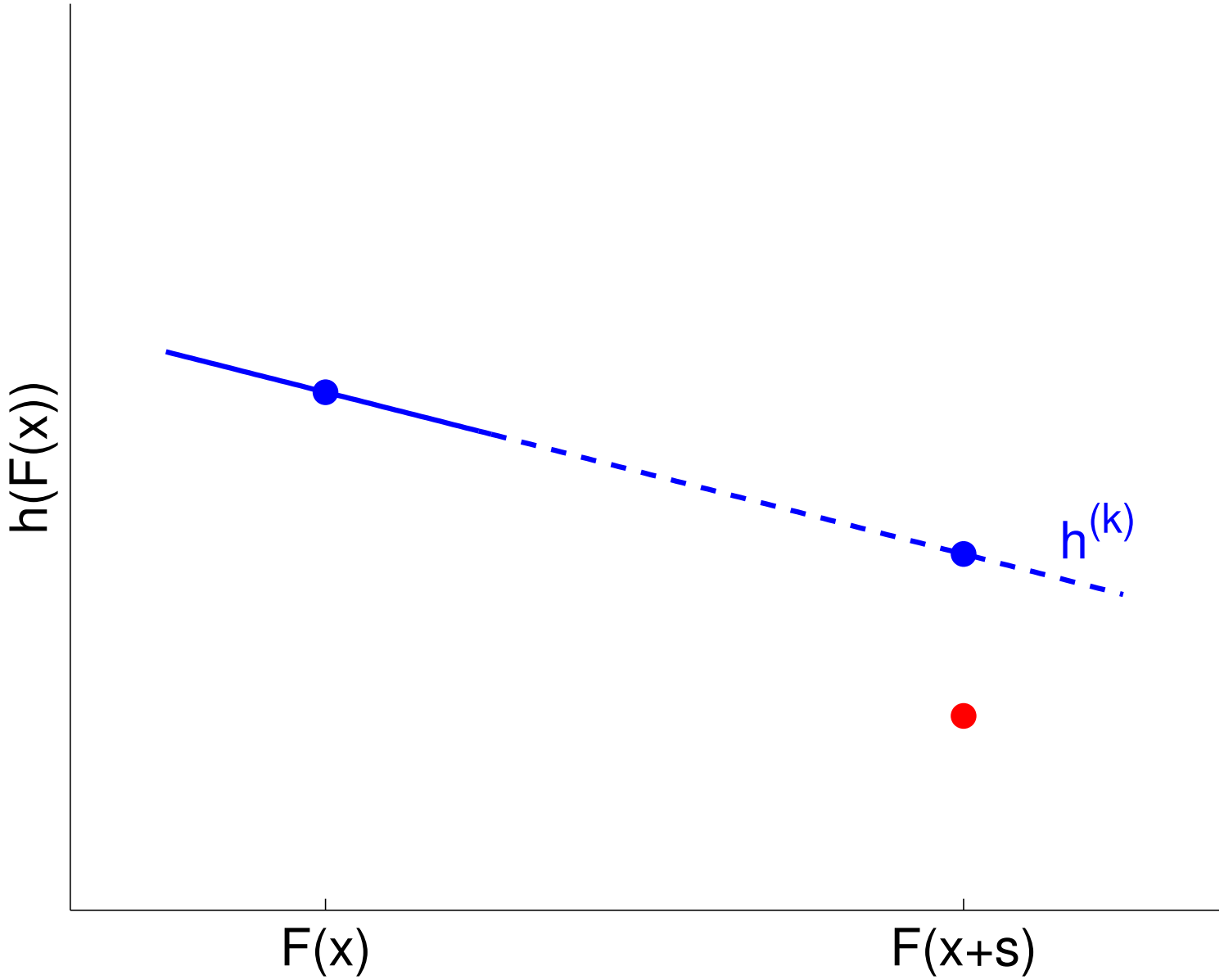

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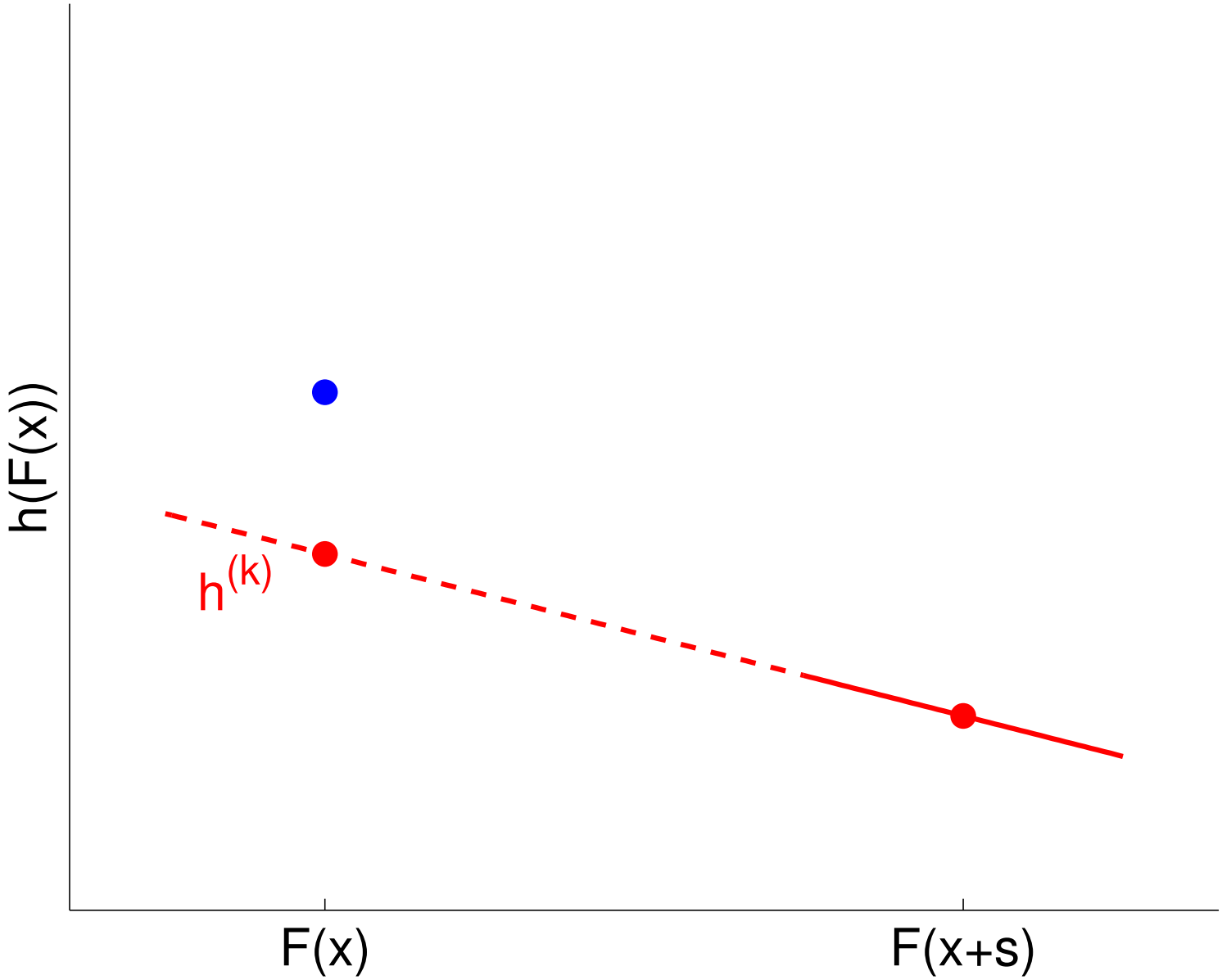

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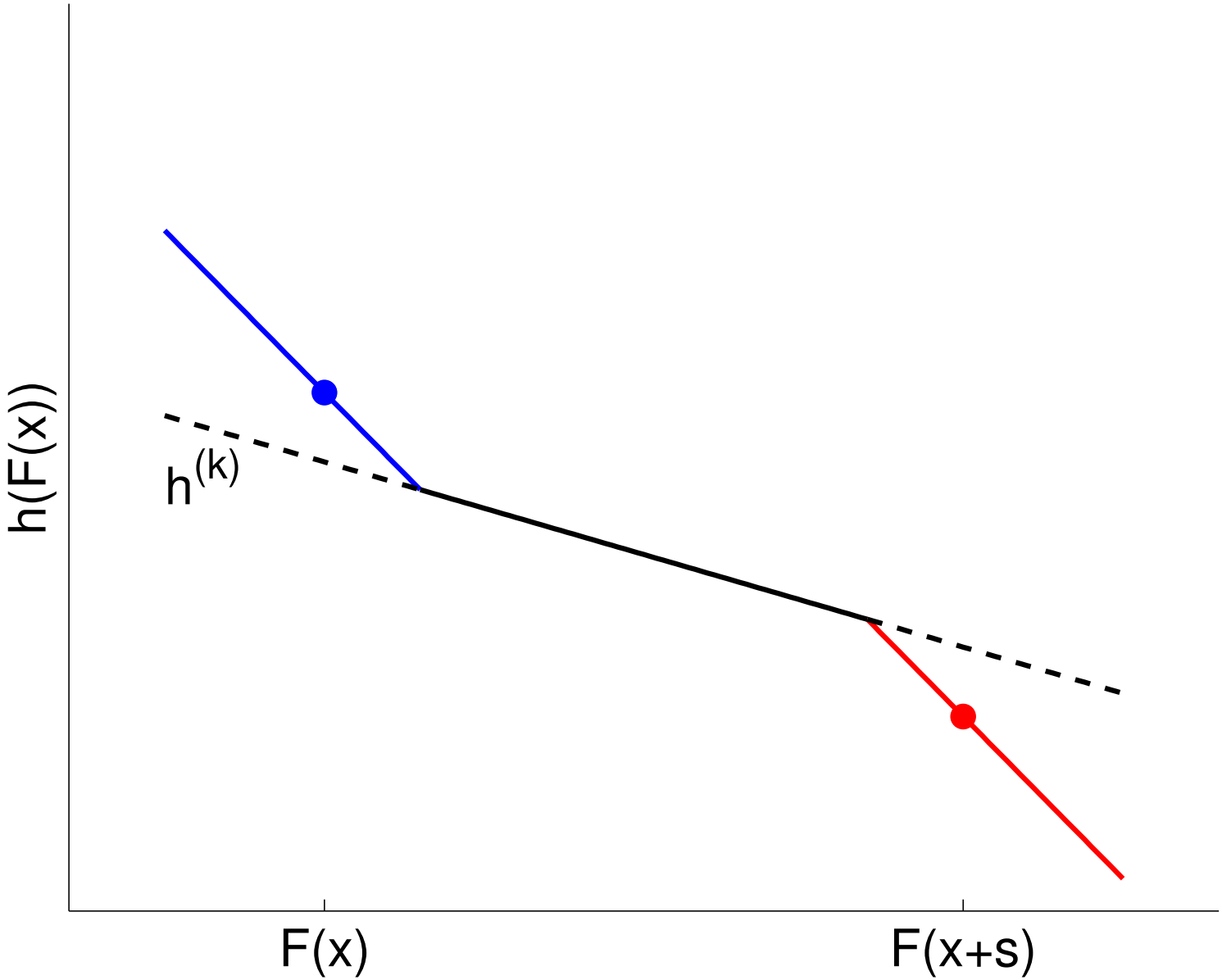

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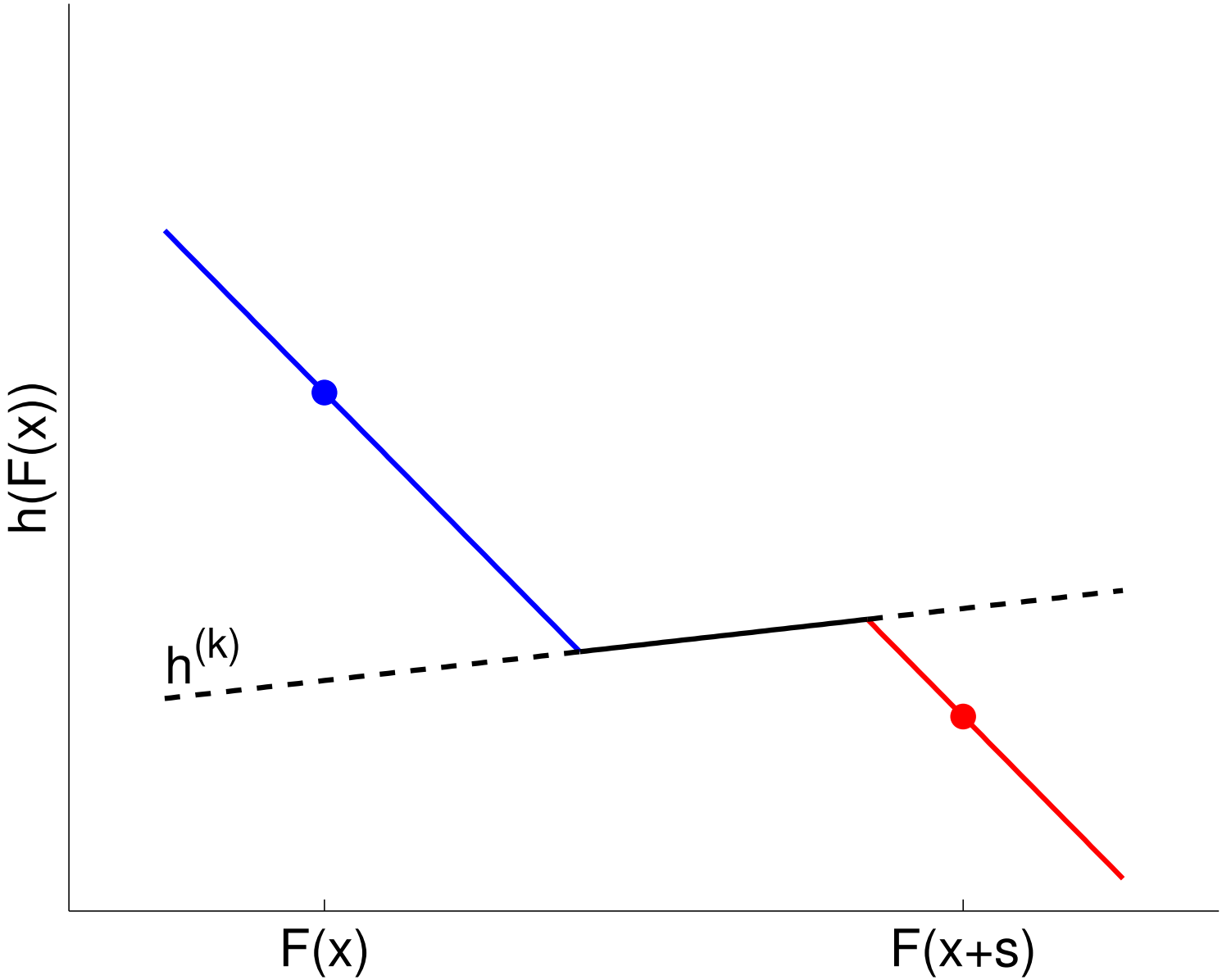

Convergence

- If the trust region radius Δ_k is a sufficiently small multiple of the model gradient $\left\|g^k\right\|$, the iteration is guaranteed to be successful.
- $\lim_{k\to\infty} \Delta_k = 0$.
- Some subsequence of master model gradients g^k goes zero.
- Zero is in the generalized Clarke subdifferential of cluster points of any subsequence of iterates with master model gradients converging to zero.
- The same holds for cluster points of the entire sequence of iterates.

Conclusions

When optimizing functions of the form $h(F(x))$ when

- h is "easy"
- F is "hard"

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Thank you!